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A CRITICISM OF EINSTEIN
AND HIS PROBLEM

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A CRITICISM OF
EINSTEIN AND
HIS PROBLEM

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OXFORD

BASIL BLACKWELL · BROAD STREET

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By the same Author :

THE MORAL SYSTEM OF DANTE'S INFERNO (1909)

AN ESSAY ON THE POLITICAL THEORY OF DANTE (1916)

THE REVOLT OF LABOUR AGAINST CIVILIZATION

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In re naturaliter obscura, qui in exponendo
plura quam necesse est superfundit, addit
tenebras, non adimit densitatem.

MACROBIUS.

CHAPTER I

THE CASE OF THE SWIMMER AND THE STREAM

A COMMON infirmity of mathematicians is failure to express themselves in a manner intelligible to the vulgar. Many books devoted to this new problem of 'relativity' have been composed with the sincere intention of avoiding technicalities; but always, after some three or four chapters, the author wanders away to the familiar mysteries, leaving his profane companions to wait outside the temple. When a mathematician declares that he is going to write for the general public, he means, apparently, that his pages will not wholly be covered with equations, and that good, sound, dictionary words will be freely employed. He forgets how deeply his own mind is saturated with assumptions to which the layman has no kind of clue. This little foible, to be sure, is not peculiar to mathematicians. Most of us who chance to have mastered a technical language are slow to unlearn it again for the benefit of others. We chafe at their innocent questions, and too hastily acquit them of brains. It is only that mathematics, more than any

other science, is forced by its nature to dwell among distant abstractions, to which the passage is too narrow and arduous to be traversed without an affable guide. The object of these few pages is to review the elements of Einstein's problem without pretence of mathematical intricacy, for which, indeed, the author can boast no qualifications. Even less will the reader find here a book of metaphysics, or an attempt to catalogue the many senses of 'relativity' which careful analysis might disclose. The most definite presupposition of the argument is a belief that difficulties, like the entities of Ockham, should not be multiplied beyond what is necessary. Without an elaborate use of symbols the manifold development of mathematical principles would doubtless be impossible; but, if the principles themselves be not amenable to simple expression, we may fairly be excused for doubting their truth.

My own interest in the subject was excited chiefly by accounts of the Michelson-Morley experiment, or rather, by the analogy, so often quoted in that context, between the experience of a swimmer in any running stream and the behaviour of light in the problematical 'ether.' On the supposition that it takes a swimmer, capable of a certain velocity in still water, a longer time to swim a measured distance, half

up and half down stream, than to swim the same distance at right angles to the current, some important conclusions were based on the failure to detect by the famous experiment any difference in velocity between two rays of light, so directed as to travel an identical distance upon two different axes, at right angles to one another. Postponing for the present all reference to the special question of light and the ether, I wish first to investigate a little more closely the variable fortunes of the swimmer in the stream. The figures justifying the ordinary doctrine may easily be found (as, for example, in the first chapter of Professor Eddington's *Time, Space and Gravitation*), nor do I propose to question their accuracy. The result, nevertheless, is a little perplexing, and one can imagine an amateurish critic, with a certain instinctive taste for probability, hankering after a very different conclusion. Given that a swimmer has a normal velocity in still water, it seems clear enough that something must be subtracted from his ordinary speed when he has to struggle against a current, and something added when he travels with its aid. But does it not also seem obvious that the stream must repay on the downward course exactly as much as it borrowed on the upward? If x be the normal velocity, and

$x - y$ the effect of retardation, must not $x + y$ be the figure of the greater velocity achieved in the downward journey? But if so, the average velocity for the whole performance will be x , which, by hypothesis, is the man's normal rate in still water. On the other hand, when he swims across the stream, there must always be some retardation, no matter how slight it may be. Hence at no time in crossing the river can he attain to the velocity x . His progress, therefore, on the up and down journey should always be better than on the other.

Now this reasoning is admirable except for one omission. It is important to note a change in the manner of reckoning, as compared with the usual way of presenting the facts. The difference is that, in one case, the trial of speeds is relative to a fixed distance in *space*, in the other, to a fixed period of *time*. If a man with a velocity in still water of one mile an hour is commanded to swim for thirty minutes against a stream that halves his velocity, he will accomplish just a quarter of a mile in the time. If he then turns about and swims for the next thirty minutes with the current to help him, his distance will be neither one quarter of a mile, nor two, but three. His total, therefore, will be exactly one mile in the hour, which

represents his normal velocity. On the other hand, when he swims across the stream in either direction, he will continuously be subject to some retardation (no matter what the precise figure may be), with the result that he never can attain to that same normal velocity, and so must cover less distance in an hour. This argument is quite unassailable; and, though it may, in some sense, be compatible with the other, it does somehow leave a different impression on the mind, and is provocative of further reflection. The same point may conveniently be illustrated from the case of a pedestrian. A man may decide one day to walk to a place four miles away on the top of a hill, and thence to return by the same route. During the outward and uphill journey he walks at only two miles an hour, and so arrives there in just two hours. Returning downhill he raises his pace to four miles an hour, and thus needs but one hour to get home. Now here, though he has travelled one half of the total distance at two miles an hour, the other half at four, his average speed has not been three miles to the hour. For he has taken three hours altogether, and the distance was only eight miles. Another day, perhaps, he decides not to walk to any fixed destination, but merely to take his exercise for two hours. In the first hour the hill is

mostly against him, and he travels no more than two miles. For the rest of his way, however, he has the ground in his favour, and in the same time covers double the distance. Two miles in the first hour, and four in the second, gives him exactly an average of three, which, perhaps, he is wont to regard as his normal velocity when walking on level ground. What answer, then, will he give to the question, whether it is easier to accomplish a journey bisected into uphill and downhill halves, or the same journey spread out on the flat? While he is standing perplexed for a moment, a smart young relativist may drop in and offer to help. 'The truth is,' he will say, 'that, on the former occasion, the first four miles of your walk contained 30 minutes apiece, the second four only 15; whereas, in the later exhibition of your velocity, the first hour consisted of only two miles, the second of four.' Admiring this ingenious 'transformation,' our pedestrian friend may still catch himself wondering how many minutes there usually are in a mile, and how many yards in an hour. But let us leave him and return to the stream.

When the swimmer's course is measured in spatial distance, his time for the two successive trials appears to vary; but when the course is a fixed period of time, and he is again asked

to compare the journey across the stream with the journey upstream and down, the variation appears in space-distance. We note, however, that he covers a *greater distance* (*i.e.* preserves a higher average velocity) when he divides the time equally between upstream and down than when he travels all the while across the current. How is this fact to be reconciled with the result of the space-course, where the cross-journey appears to take *less time* than the other? But first let us try a third experiment. Having laid the course, first in space, then in time, let us try the effect of laying it in velocity. Here, perhaps, we may find ourselves puzzled. Comparison of velocities has, hitherto, been the whole object of the swimming performance. How then, if the velocity is to be invariable, can the problem be said to exist? In itself, however, there is nothing improbable in the thought of a human swimmer who can preserve the same velocity at various angles to the force of the stream. Usually, it is true, a man will diminish his normal speed, as measured in a tank or pond, when he has to swim in the teeth of a current. But when called upon to swim a *race* of, say, 100 yards against a moderate stream, he may well succeed, by dint of special effort, in maintaining his pond-velocity over the course. So too, when the race is finished, and

he turns to drift back to the starting-point, he will leave the stream to do most of the work for him, and thus once more may travel with his average speed. Human beings, however, are incalculable creatures, and psychological causes had best be excluded from physics. Suppose, then, you were to hear of a mechanical swimmer, which travelled up, down, or across any current with unaltered velocity. At first you would refuse to believe it, but the testimony of impeccable witnesses might induce you to look into the matter. Having failed to convict the automaton of trickery, or to detect any error in the measurements of velocity, you would cast about next for some rational explanation. The hypothesis of a new and mysterious 'force' might tempt you, but we at least, with a view to the character of our whole enquiry, must firmly expel that god from the machine. There is no room for 'force' in the argument, if this third trial is to be comparable with the other two. What possible explanation, then, could be offered of a velocity unaffected by change of direction in a stream running 5 or 10 miles an hour? Two, and only two (we may think at first) could account for the phenomenon, and both of them sound absurd.

In the first place, you might lose your temper

and protest that the thing did not move at all. But if this seemed too flatly opposed to the evidence, the only remaining alternative would be to infer that time and space themselves must shrink and expand. Remote from each other as these two suggestions must seem, we shall find occasion, as the argument proceeds, to examine them both. We shall have to enquire whether so queer a dilemma can actually be forced on us, and even whether it is necessary to destroy it by accepting both horns at once. At this stage, however, it is only the second of the two explanations that calls for attention. We have noticed already, when reflecting on the swimmer and the walker, the alternate variation of time and of space. When the swimmer's distance was the same in the two courses, his times were different; when the period of time was fixed, his distances were unequal; and in either case, of course, his velocity was affected. But since the compound of time and space is velocity, is there not a third possibility, that, when his velocity is constant, the *actual* times and *actual* distances must be the variable factors? Are the mile and the hour quite the stolid old conservatives we have always supposed them to be? Observe, too, that it would be useless to endow only one of them with elastic dimensions; for

then the velocity might not always be constant. The variation must be possible in either. But where can we find a swimmer with constant velocity in all circumstances? We shall not have far to go.

The conventional doctrine, that less time is required to swim 100 yards across a river than to swim 50 upstream and then 50 down, must credit the swimmer with a certain standard velocity in still water, and this he must be supposed to retain. In other words, all the subsequent calculations depend upon the assumption that a certain normal speed is diminished or augmented by the current in which he is to swim. Without this the whole problem collapses. So many yards a minute (accomplished by so many similar strokes in motionless water) must be regarded as his private possession, or there is no sense at all in proceeding to calculate the varying effects of the stream. Now, when this much is conceded, and when it is found that the man, swimming the same distance twice, spends more time on one course than on the other, only one explanation is possible without falling into open absurdity. The normal velocity is, by hypothesis, unaltered; the times are uneven; the distances, therefore, are *not* the same. I have no thought of impugning the accuracy of

the space-measurements made on the bank, or the respectability of the watches used for the timing. The precision of these I take to be absolute, but I submit that the swimmer is subjected to two unequal trials. He is asked to swim, first a shorter, then a longer course, as though their length were the same. A more scientific way, perhaps, of expressing my proposition would be to assert at once that 'retardation' is a myth. But as that might be rather too sudden a shock, we must turn aside for some preliminary reflections.

Every swimmer, like every oarsman, will assure you that going upstream is harder work than coming down. Every swimmer, therefore, and every oarsman will be wrong. There is no difference whatever between the two processes, so long as you stick to the relevant facts. The common belief depends primarily upon the ambition to reach a certain point on the *bank*. It depends also on physiological or psychological causes, which have nothing to do with the question before us. When we laugh at the thought of a swimmer who always travels exactly the same distance in a given time, regardless of current and direction, this is chiefly because we insist on measuring the distance by the irrelevant bank, instead of by the water itself; and again, because we remember our

physical exhaustion after battling against the stream. But why this superfluous battle? Simply because we were aiming at some point on the bank. Abandon this idle preference; be content to make your normal number of normal strokes in a minute, and the measurable result of them, expressed in what we may call water-distance, will never vary so much as the breadth of a hair. There is nothing mysterious about 'water-distance'; it is just the same as distance on land. I introduce the term only as a gentle reminder that swimming is usually done in the water, not on the bank. As a simple illustration of the point, place a long stick in a river, arrange that no waves or eddies shall disturb its orderly progress, and allow it to float in a line with the current. It will then be, for the purposes of the experiment, an integral part of the river, with the river's exact velocity and no other. Now place a swimmer at one end of it, and bid him swim with his normal effort (*i.e.* that which produces his velocity in still water) to the other end. Time him carefully, and then repeat the trial in the opposite direction, stipulating that he shall make exactly the same number of strokes, with exactly the same vigour, as before. Beyond a shadow of doubt, he will accomplish his task in exactly the same time as before; and this he

will do over and over again, at whatever angle to the current you choose to arrange the stick, and no matter what comments may be offered by loafers on the bank. In actual rivers, no doubt, it would be difficult to secure the required conditions for a satisfactory experiment with a stick; but, in effect, this is what every swimmer does in the test so freely quoted in connection with the Michelson-Morley experiment. If, then, he takes more time to go upstream than to return to the same point on the bank, it can only be because he swims beyond the end of the stick before he returns. Similarly, if the time across the stream (*i.e.* once across) is shorter than on the upward course, but longer than on the downward, the corresponding ratio between the distances must hold good. And this it is perfectly easy to prove in an analogous case to which I shall shortly proceed.

The sensation of effort when one struggles against a current of water, an adverse wind, or, for that matter, the gradient of a hill, is so familiar that the theory of 'retardation' and 'resistance' will not easily be abandoned. Imagine, however, the plight of a swimmer set in the midst of an ocean on a night of absolute and impenetrable darkness. No friendly bank is there to arouse his longing, no pole-star to

guide him by its beams. Better still would it be if, like a fish, he could swim under water. But if he is to remain with his head on the surface, not even the buffeting waves must be allowed to give him a hint. All must be dark and sleek and oily, as he cleaves his lonely way. Man is vexed with imagination, and I can easily credit such a swimmer with various emotions. He may even begin to wonder how long it will be before he must permanently cast in his lot with the fish. But I challenge him to have the slightest sense of direction; I challenge him to find it easier or harder to go one way than another, or (unless it be from the rush of the air) to guess that he is being swept along northwards, perhaps, or southwards with a velocity of 20 miles an hour. Should he succeed in persuading himself that one route is easier than another, this will be merely the work of fancy; and, while he supposes himself to be moving steadily in one direction, he will almost certainly be swimming round and round the circumference of a circle. Precisely the same would be the case of the swimmer between ordinary banks, could he but banish his preference for some particular direction, his querulous anxiety to reach a point on the shore. All this I set down in the fond hope of diminishing initial prejudice against the reception of an

elementary truth. Strictly speaking, the psychological aspect is irrelevant to the physical problem; and in the next chapter the proof that a swimmer obliged to conform to a rule imposed by the bank does swim further upstream than down will be set forth without reference to the swimmer's emotions.

CHAPTER II

THE CASE OF THE PASSENGER AND THE TRAIN

To Einstein belongs the credit of raising an important question, even if it is difficult to believe that he has given the right answer. In his valuable little book, translated by Dr. Lawson, of Sheffield University, he discusses the case (§§ 6, 9 and 10) of a man who walks along the corridor of a moving train, in the direction of the engine (though the direction is not here important), and covers so many yards in a certain number of seconds. The question is whether the measurements of time and space made within the train can be simply identified with the like measurements made on the embankment. The man walks, perhaps, at the rate of 2 yards per second within the train. Does it follow that he is likewise walking at 2 yards per second as judged by and from the embankment? If the 2 yards be accepted as the space-distance, does it follow that these yards are covered in what we may call 1 embankment second? Or again, if we take 1 second as the given time, does it follow that the distance covered is equal to 2 embankment

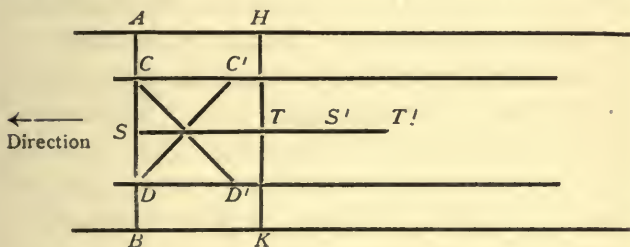
yards? Einstein's answer is negative. He discovers here the relativity of both time and distance, and forthwith sweeps us away to the 'Lorentz transformation' and to speculations about the mysterious behaviour of clocks and measuring-rods. All this I believe to be entirely superfluous, because it all rests on a false presupposition, derived from, or akin to, the error made in the Michelson-Morley experiment, or rather in the interpretation of the case of the swimmer.

We have first to consider, as Einstein himself has remarked, that a passenger walking up, down or across the corridor of a moving train is exactly analogous to a man swimming up, down or across a flowing stream. The banks are the same in both cases, the passenger is the swimmer, the train is the stream. What Einstein, so far as I can judge, has failed to perceive is the solution of the swimming problem thus plainly afforded. When the passenger walks towards the tail of the train, that is to say, in the opposite direction to that of the train, as judged from the bank, he is strictly analogous to the man swimming upstream. When he reverses his direction, he is walking downstream, and, when he walks to and fro across the corridor or carriage, he is swimming across the current. Why, then, does he lack

the swimmer's varying sensations? Solely because he is not trying to reach a point on the *bank*. He is travelling, perhaps, in the carriage nearest the engine, and the restaurant-car is at the tail of the train. In due course he desires his luncheon, and walks along the corridor to get it. While he walks at his normal 3 miles an hour in one direction, it worries him not at all if the train is carrying him at, perhaps, 50 or 60 miles an hour in the opposite direction. He is not thinking about the embankment, but is brooding on his imminent lunch. Nor, I suppose, will anyone dream of disputing his ability to walk with the same velocity, without varying his effort, no matter what his direction within the train. In point of fact, he can easily get sensations analogous to those of swimming upstream, if he tries to return to a point on the embankment already passed by the train. To do this would often be impossible, but, if the train happened to be moving at only some 5 or 6 miles an hour, it could be done with an effort, that is to say, by walking or running with more than the passenger's normal velocity.

Let us proceed, then, to examine the case of the passenger with the help of a diagram. Trains usually go faster than men, nor does the speed of the train affect the principle of the

argument; but, for the sake of a closer analogy to a swimmer who can make some actual progress upstream, as judged from the bank, I shall slow down the train till it is crawling along at 3 yards a second, while the passenger shall be hurried up to 5 yards a second; so that, were he to alight and walk side by side with the train, he would gain on it at the rate of 2 yards a second. Take the following diagram :



The outer pair of parallel lines represent the embankment, the inner pair the train. The line AB is drawn at right angles to the embankment and the train; CD is the interior width of the carriage; AH , ST and BK are each equal to CD . With these data in his possession, as well as the actual velocity of the train and the normal velocity of the passenger 'in still water,' the observer on the bank sets out to enquire how long it will take the passenger, first to walk along CD and back to C , then to

walk from S to T , and back once more to S . Arguing on the basis of the 'retardation' theory, the observer will calculate that more time will be required to walk from C to D while the train is moving than when it is at rest; for by CD he will mean a certain portion of the straight line AB , and the train, he will argue, must all the while be carrying the passenger 'downstream.' Referring to his data, he will come to the conclusion that the man's velocity along CD or DC will be reduced from 5 yards a second to 4. Similarly, he will calculate the 'upstream' velocity as 2 yards per second, the 'downstream' as 8. But here I must interpolate an important note on the general character of the argument, namely, that it makes not the slightest difference whether the figures calculated for the retardation and acceleration are accurate or not. That is to say, it makes no difference in *principle*, however much it may affect the value of the result in a particular case. Instead of my figures 4, 2 and 8 yards per second, any others may be taken—the more absurd the better—to represent the different velocities, without touching the principle of the argument by which this problem of 'relativity' is solved. Well then, to return to the case before us, we must now name a definite figure for the length

of CD . Four yards will be the most convenient, because that is the distance, according to the embankment calculation, to be accomplished in one second by the passenger when he is walking across the current of the train. The passenger will thus be allowed two seconds to walk from C to D and back again to C . But now the fun begins. For the passenger walks at 5 yards per second in any direction without the smallest difficulty, and is forbidden by his principles to depart from his standard velocity. To put it another way, he *must* walk 5 yards, not 4, if he is to remain on the line CD , as viewed from the embankment, so as to arrive at D at the end of one second. His course, therefore, in the diagram must be CD' in one direction, DC' in the other, and the length of each of those lines will be 5 yards. Further, since each is the hypotenuse of a right-angled triangle, the length of the remaining side (CC' or DD') will be 3 yards, by *Euclid* I., 47.

Our passenger, then, walks 10 yards in the first 2 seconds, while the scientific gentlemen on the bank are prepared to swear that he has walked only 8. Next comes the journey from S to T . According to the bank calculation, the man will walk this 4 yards at only 2 yards per second, and so will require 2 seconds for his task. But during this time the passenger,

who knows nothing of 'retardation' will walk 10 yards up the train, which gives us the length of ST' . At T' he will appear to them to be standing at T , and his last stage, according to them, will be to return from T to S with a speed of 8 yards per second. For this they will allow him just half a second, but he, as before, will preserve his constant velocity, and will travel only $2\frac{1}{2}$ yards in the time; and thus we get the length of $T'S'$. Both parties now proceed to add up their totals. On the bank they find that the man has walked 16 yards in $4\frac{1}{2}$ seconds, and that it took him $2\frac{1}{2}$ seconds to do the 8 yards up train and down, as against 2 seconds for the same distance across the stream and back. He, on the other hand, knows that in their $4\frac{1}{2}$ seconds he has walked $22\frac{1}{2}$ yards, and that his four stages, in yards, were 5, 5, 10, and $2\frac{1}{2}$. The number of yards walked by the passenger, in this particular example, must always be 5 times the number of seconds calculated on the bank. Had his velocity been 6, 10, 100, or any other number of yards per second we should have had to multiply their number of seconds by his actual number of yards per second, instead of by 5, but the principle would be always the same. The velocity of the train in no way affects *his* velocity. There is neither retardation nor

acceleration, and exactly the same is true of the swimmer in the stream. He merely swims greater or smaller distances, according to the number of seconds he occupies in swimming.

Perhaps it will be useful to exhibit the principle in the alternative way. In the first trial a certain number of seconds were, so to say, allotted to the passenger, and he translated them into his own number of yards. But now suppose they ask him to walk 4 yards across the train, 4 back again, 4 up-train, and 4 down. As before they will calculate his time as $4\frac{1}{2}$ seconds; but he, feeling bound on this occasion to stick to their distances, will only require $3\frac{1}{5}$ seconds for the whole journey; the number of their yards in this case being 5 times the number of his seconds. Nothing new is revealed in this second presentation of the facts. The only difference is that the track is measured in time on one occasion, in space on the other. In both cases, the philosophers of the embankment, with all the requisite data at their disposal, fell into the same error, because they would talk about 'retardation,' instead of grasping the cardinal fact, that the velocity of the train could have no possible effect on the constant velocity of the passenger, as he walks to and fro within the train.

There still remains the last, and not the least

instructive, of the three possibilities, when the track is, so to speak, laid in velocity, not in space or in time. The observers on the bank, we now assume, know the passenger's velocity, and have somehow persuaded themselves that it remains constant in all circumstances. Their object is to discover the width of the carriage (that is to say, the length of CD) and the length of time required for crossing it. The time they hope to discover by observation, and thence to deduce the distance between C and D . Keeping a sharp watch, and choosing a favourable occasion, they observe him leave C and walk along the line to D , which they, of course, see as part of the line AB . After exactly one second he is observed to arrive at D , whence they infer that the distance must be 5 yards. But though it is true that he has walked 5 yards in one second, his actual course has been from C to D' . They have thus provided themselves with a false measuring-rod, and therewith will roam about the world computing times and distances and getting them wrong every time. For they will measure with their own CD , a distance of 4 yards, which the passenger would traverse in $\frac{4}{5}$ of a second, and will count it as 5 yards, or, expressed in time, as one second. Other varieties of this error could be constructed, but the general principle

will always be the same. In all cases, whether they start from a known distance between two points or from the observation of a time actually occupied by the passenger in covering a distance not yet measured, their genuine knowledge of his velocity will be the very cause of their subsequent errors.

Reviewing the results of the three alternative trials, we note that three factors are involved, distance in space, distance in time, and velocity; and we find that, whenever the bank party have correct information about any one of the three, they must go astray about the other two. Yet all three varieties of the error spring from a common source, their failure to detect the actual course of the traveller. D and D' are points in the train 3 yards apart. There is nothing mysterious about them; they are not one confused identity, but separate points which might be marked with chalk and observed by anybody who took the trouble to visit them. The passenger goes always to one; the observers suppose him to go always to the other. Similarly, his actual return across the carriage is always at the angle represented by DC' in the diagram; and again, when they see him at T he is at T' , and, when they believe him to have returned to S , he has stopped short at S' . Exactly the same things

happen in the case of the swimmer in the stream. The spectators see him swim across a straight line from bank to bank, but in order to afford them that spectacle, he has to swim aslant across the water. He actually swims along the hypotenuse of a right-angled triangle, while they firmly believe him to be travelling along one of the other sides. There is no 'retardation' when he goes across or up the stream, no 'acceleration' or additional velocity when he comes down with the current. He keeps always to his constant and uniform velocity, just as though he were disporting himself in his private tank.

Consideration of these facts, which appear to be indisputable, sets us wondering whether the whole imposing fabric of 'relativity' is not already tottering rather ominously on its base. Further light will be thrown on that question in the next chapter. Meanwhile our attention has been drawn to a very important fact, which might provisionally be called 'relativity of direction.' For since the passenger can only walk along the line CD , regarded as part of the line AB , by actually walking along the different line CD' , we have the materials for a very pretty dispute about 'straight lines' and 'shortest distances.' This subject, however, is reserved for a later chapter, where its full significance will be more clearly discerned.

CHAPTER III

THE ADDITION OF VELOCITIES

IN THE sixth section of his book Einstein states the classical theorem of the 'addition of velocities,' and in due course proceeds to question its truth. His most important argument refers to the constant velocity of light and Fizeau's experiment, but this it will be expedient to postpone until we have examined the simpler, but supposedly analogous, case of a man walking along the corridor of a train in the direction of the train's journey, that is to say, towards the engine. The train, perhaps, is travelling at 60 miles an hour, and the man is walking at 3. Hence, according to the classical doctrine, he ought to be advancing at the rate of 63 miles an hour relatively to the embankment. But this, Einstein proceeds to argue, is an untenable conclusion, and consequently the classical theorem can no longer be upheld. We thus resume the enquiry stated already at the beginning of the preceding chapter, whether times and distances, as measured within the train, are absolutely the same as when the embankment is the reference-

body, or whether a 'transformation' is required.

Here, to begin with, there seems to be a surprising confusion between two distinct questions, the one relating to the actual point on the embankment at which the passenger arrives in a given time, the other to the addition of velocities. The answer to the first presents no difficulties, and indeed admits of no doubt; the answer to the second, or at least the first part of the answer, is that 'addition of velocities' is an extremely careless and uncritical expression. The easier point can be determined by the following example. Before the train leaves London a chalk line is drawn across the corridor, at right angles to the sides of the train. This line is extended to a point *A* on the embankment (or platform), which it meets at right angles. We thus have *A* marked as the starting-point of both the train and the man. We place the man firmly on the chalk line, and start the train. In an hour the train travels 60 miles and stops. Meanwhile, at some time during the journey, no matter when, the man walks 20 yards towards the engine, chalks another line parallel to the first, and stands on it till the train comes to rest. When the train stops, the original and the later chalk lines are extended, respectively, to points *B*

and C on the embankment. The train, therefore, as represented by the first line, has travelled a distance AB , which is 60 miles, and the man has arrived at C . What is the length of BC ? The only conceivable answer is 20 yards, and the fact can be tested any day of the week. Now, if the train's journey had been shorter, or the man's walk in the train much longer, we might have persuaded him to move continuously, and with uniform velocity, so as to arrive at the second line exactly when the train finished its journey. The result, of course, as regards the eventual distance from A (and from B) is just the same in principle whether he walks throughout the train's journey or only during some part of it; nor would there be any change in the nature of the problem, if the direction of the walk were towards the tail of the train. There is even a sense (an important sense, as we shall presently see) in which the man need never have been in the train at all. We might have placed him at B , with orders to get to C when and how he pleased, so long as he arrived before the train pulled up at B . Any spare time he could then have occupied in wondering whether Achilles would ever catch the tortoise, if the hero were obliged to *stop* every time his rival felt disposed for a rest.

Thus far we have dealt only with the obvious. But now to the problem of velocities. And here we stumble at once on the clumsy use of the term 'addition of velocities,' which seems to have gained such unfortunate currency. What justification can there be for saying that *A*'s velocity is added to *B*'s, unless *B*'s is at the same time added to *A*'s? If there be any way of joining a velocity of 30 miles to another like velocity, so that the net result is a single velocity of 60 miles an hour, two velocities will then, in a reasonable sense, be added together; just as 3 is added to 2 by the same act that adds 2 to 3. But there is no analogy to this in the case of the man and the train. There is no addition of velocities. For no one, I presume, is likely to suggest that, while the man is walking at 3 miles an hour towards the engine, the velocity of the train is raised from 60 to 63; or that, if he chooses to walk in the opposite direction, the speed of the train is reduced to 57? If not, it is more important to attack this indefensible use of the term 'addition' than to raise doubts about the classical theorem. My objection is not to be dismissed as pedantic. Even within the territory of science convention may excuse much doubtful usage of words; but there are times when acquiescence in convention may engender con-

fusion of thought. So it is in the example before us, for it looks as though 'addition of velocities' had brought Einstein himself to grief.

Let us scrutinise the facts a little more closely. One point, to begin with, is indisputable, and may as well be set down at once. If you choose to attend only to the bare fact that the man (according to the figures we have used) has arrived, in the course of one hour, at a point just 60 miles and 20 yards from where he started, you are then fully entitled to say that he passed from *A* to *C* with a velocity of 60 miles 20 yards to the hour. Had the train been 3 miles long, and had he walked with uniform velocity throughout the hour, so as exactly to finish the 3 miles as the train came to rest, nothing can alter the fact that his velocity (from our present point of view) would have been an unbroken 63 miles an hour, as measured by the embankment. Or again, had he walked the same distance in the train, but from the engine towards the tail of the train, his velocity, in the same sense, would have been 57 miles an hour; for the 57 would have been a verifiable fact, and would have meant simply his distance from the starting-point at the end of one hour. The situation, then, can be summarised as follows: (1) In one hour the

man has gone so many miles + (or —) so many yards from the starting-point; (2) those yards were walked on a motionless body, the train, and were measured in the train; (3) at the end of the journey their number and length is tested by another motionless body, the embankment, and the agreement is precise; (4) the man walked those yards at a uniform rate (*e.g.* 3 miles an hour) as timed in the train, with a result exactly the same as if he had walked them with that velocity on the embankment in accordance with embankment-time. These four propositions state ascertainable, verifiable facts. In face of them, we are asked to believe that there is, nevertheless, a mysterious difference between the length of the train, or the length of the yards, while the train and the passenger are in motion, and the length of the same train and yards when the train is at rest. This, I submit, sounds more like hocus-pocus than science, and is faintly suggestive of rabbits emerging from hats.

But now let us inspect the facts from another and, I venture to think, a more intelligent point of view. What was the velocity of the passenger during his promenade in the train? The right answer, the most scientific answer, is 3 miles an hour. This answer, moreover, is true in exactly the same sense relatively to

the train, the embankment, the sun, or any other body you choose to take, so long as you do not confound together different kinds of relation. It is the constant velocity of the passenger in the motionless train, like the constant velocity of the swimmer in the motionless stream. The velocity of the man and the velocity of the train were never added; they were two facts as independent of one another as if the train had been in Sirius, the man in Regent's Park. You can, if you please, add together the distances travelled by each in a given time; but a great many other distances were being travelled in that same time, if you choose to look about the world and collect them. The moon was revolving in its orbit, and an old gentleman, perhaps, was falling downstairs. To a critical eye those two incidents were not more distinct than the velocities of the man and the train. The choice lies between two alternatives: (1) the classical 'addition of velocities,' which, on its own limited ground, is quite inexpugnable; (2) complete separation of the man's velocity from the train's. There is, indeed, another way of looking at the facts, connected with what I have called 'relativity of direction'; but this, as I shall argue in another place, Einstein has altogether failed to discuss.

Let us return for a while to the swimmer. When one swimming with the stream contributes 2 miles an hour as his own velocity, and receives a like contribution from the current, does anyone propose to deny that he is carried along the bank at 4 miles an hour? The fact can be established with no more inaccuracy than is involved in the use of a stop-watch as you walk along by his side. A more interesting question, however, confronts us if we proceed to ask *where* he is swimming at 4 miles an hour. The only correct answer is, *nowhere*. Had we to choose between the two replies, 'in the water' and 'on the bank,' the latter would be the less misleading of the two. In the water he is swimming at 2 miles an hour, and would continue to do so (provided he did not vary his effort) if you were to turn him round and direct him upstream. The absurdity of trying to add his velocity to the stream's is exposed as soon as it occurs to us that his swimming at 2 miles an hour, or with any other velocity, is possible only because, relatively to his swimming, the stream is *at rest*. Swimming in a stream is a process identical in principle with walking on a road; the motion is possible, in the one case, so far as the stream does not swim; in the other, so far as the road does not walk. And since the

banks of a stream appear to be rather confusing to the intellect, let us glance for a moment at the problem of the road. When a man is travelling along the king's highway at 3 miles an hour, would you say that he was walking 3 miles an hour faster than the road? Probably not, but assuredly you *ought* to, if you propose to add a swimmer's velocity to the river's. For the road is travelling at some 30 kilometres a second (so they tell us) on its annual journey, precisely as the stream is flowing at 2 miles an hour towards the sea. So, too, if you believe that it is harder to swim upstream than down, you are bound in honour to believe that it is harder to walk, as it were, against the earth's orbit than with it. As things are, however, it will probably be allowed that '3 miles an hour' is an unmeaning expression, except in so far as the earth is at rest. A man does not race with milestones and defeat each one of them by a mile in every 20 minutes. The velocity of a milestone is zero; it does not compete. For exactly the same reason, the velocity of the passenger within the train means nothing except in so far as the train is at rest. It is, therefore, exactly the same velocity, in every relevant sense, as if it took place on the embankment, which is also assumed to be at rest. If you will not accept

this view of the situation, the only alternative is to return to the 'addition of velocities.' This is a slightly ridiculous doctrine, because there is *no* addition of velocities. There is only an addition of distances travelled by two bodies during the same period of time; after which you proceed to impute the whole of the *two* distances to only *one* of the travellers, and thence to make up a new velocity, which you likewise assign to him alone. But this is utterly irrational. If the man is entitled to the train's contribution, the train is equally entitled to the man's. The train is at rest, and the man walks on it at 3 miles an hour; the earth is at rest, and the train runs on it at 60 miles an hour. Again; the train is in motion, and the man is carried along as part of it at 60 miles an hour; the earth is in motion, and the train (including the man) is carried along by it at 30 kilometres a second. Now, when the man, after being carried 60 miles in an hour by the train, alights from his carriage and, in the next hour, proceeds to walk three miles, no one proposes to add up velocities or to enter the mazes of 'relativity.' But as soon as he walks his three miles, or some part of it, *during* his journey in the train, the whole of Europe is convulsed with equations. But why? Whether he walks in the train or on the embankment, whether he walks his miles yesterday, to-day

or to-morrow, makes not an atom of difference to the situation. You can add the train's distance and the man's distance together, if you like, and (quite illogically) attribute it all to the man. To save your reputation for clear thinking you ought, in that case, to add together also the two times. The train ran for an hour and did 60 miles; the man walked (let us now suppose) for 10 minutes and did half a mile. If you want to make up a composite velocity, the only reasonable one is $60\frac{1}{2}$ miles divided by 70 minutes. Still, the classical 'addition of velocities,' though arbitrary and rather foolish, is intelligible from a certain point of view. The only thing that seems at present to have no *locus standi* is the problem of 'relativity.'

Let us arrange a simultaneous exhibition of velocities on a somewhat larger scale. The earth flies at some 67,500 miles an hour along the motionless embankment of the sun; a train runs at 60 miles an hour along the face of the motionless earth; a long slender perambulator is pushed at 4 miles an hour along the corridor of the motionless train; an infant crawls solemnly at 1 mile an hour along the motionless perambulator; a tortoise (who cannot think in hours) marches at 12 inches a minute along the back of the motionless infant; and finally, a snail slides at 6 inches a minute

along the shell of the motionless tortoise. All face in the same direction; all are journeying from Somewhere to Nowhere; it is the day of the great Utopia Stakes. The flag falls, and off they go. At the end of one minute (for it is a time-race) the snail has its proboscis just beyond the snout of the tortoise; the tortoise is peeping over the head of the infant; the infant is gazing on the floor just ahead of his unusually long perambulator; the perambulator has the train beaten by rather more than 117 yards, and the poor old earth 'also ran.' Now for the addition of velocities. A good many sums await us, but, as their character is simple, we will send over to the nearest elementary school and enlist the aid of two or three children. In one minute, it appears, the earth has travelled about 1,125 miles, the train 1 mile, the perambulator a little more than 117 yards, the precocious infant more than 29 yards, the tortoise 12 inches, the snail exactly 6. Adding up the figures, with some disregard of fractions, we find that the snail has travelled with a velocity of 1,126 miles, 146½ yards per minute; and this, when we consider the creature's ancestry and literary reputation, is by no means a bad performance. The children will be ready to work out the figures for the remaining competitors, but, if you ask them to solve the problem of relativity, they

will pack up their pencils and run away to play in the yard. Their innocent minds can detect no such problem, and their innocent minds are right. That problem is a figment of the brain. In justice, however, to my own argument as a whole, and in justice to the general notion of relativity, it is necessary to add that in this chapter I have deliberately suppressed all reference to another aspect of the question, which cannot appear until the limitations of Euclid's geometry have been carefully examined. As Einstein says, the doctrine of the 'addition of velocities' is bound up with the acceptance of that geometry. What Einstein, I venture to think, does not see, is the nature of the alteration required when we frankly abandon Euclid for something else. His own problem of relativity, so far as it arises in connection with the 'addition of velocities,' rests on a misconception of the behaviour of swimmers and passengers, on failure to set different velocities in their proper relations, and, above all, on failure to submit the expression 'constant velocity' to critical examination. It should be superfluous to add that there is no real question of choosing between relative and absolute time and space. The sole question is what kind of relations you propose to take into account. To talk of 'absolute' time or space is to talk at random.

CHAPTER IV

LIGHT AND THE FIRST PRINCIPLE OF MOTION

WE SHALL now have to venture into deeper waters.

O voi che siete in piccioletta barca,
desiderosi d' ascoltar seguiti
retro al mio legno che cantando varca,
Tornate a riveder li vostri liti.

True, we have not yet had much time for singing, but Dante's word of warning, as he stepped into the kingdom of light, still deserves our most serious attention. For in Einstein's doctrine, or rather, in the first part of it, the constant velocity of light is the pivot upon which the whole of relativity turns. To retain that constancy, that famous 300,000 kilometres per second, and at the same time to make all other velocities relative, is an intellectual feat of surprising agility. In what sense it is possible, and what is its ultimate meaning, I must now attempt to shew.

Einstein's criticism of the classical 'addition of velocities' finds a special application to the velocity of light in the celebrated experiment of Fizeau, performed more than fifty years ago.

This is discussed in § 13 of Einstein's book, and another illustration of the same point, rather less easy to follow, is given in § 7. His object is, once more, to construct an analogy to the passenger, the train and the bank. In Fizeau's experiment, a tube is the embankment, a liquid moving through the tube is the train, and a ray of light passed through the liquid is the passenger. In the other example, the train and the embankment are retained in their proper form, a ray of light is sent along the embankment, and the question is, how fast does the light travel in relation to the moving train? Now, according to the classical doctrine of the addition of velocities (so, at least, Einstein implies), the speed of the light, relatively to the train, should be less than c (the symbol for its constant velocity *in vacuo*), because the train is moving along the embankment, with its own velocity, in the same direction as the travelling ray. But here there seems to be a rather fatal confusion about the 'addition of velocities.' In the first place, if the ray is to be analogous to a passenger, we ought (by the classical theorem) to argue that its velocity, in relation to the embankment, is greater than c , just as the human passenger walking along the corridor of a 60 miles an hour train, towards the engine, with his own

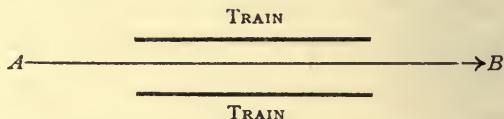
velocity of 3 miles an hour, is whirled along the embankment at the rate of $60 + 3$. By a curious accident, however, Einstein has forgotten to put his light-passenger in the train, and has left it to course along the embankment with its own admirable velocity. The result is a little bewildering. For (1) relatively to the embankment (or rather, along the embankment) the ray is travelling with its usual c ; (2) still on the embankment, but relatively to the train, it is travelling with a velocity less than c , because the train is running after it at 60 miles an hour; (3) in its character as a passenger—morally, though not physically, inside the train—it ought to be travelling with a velocity of $c + 60$ miles an hour relatively to the embankment; (4) still in its character of passenger, it can doubtless walk up and down or across the train with its own unalterable c . What a complicated journey it is! Let us see whether these various phases in the behaviour of the ray can be sorted out and critically examined.

So far as the classial theorem for the 'addition of velocities' only means that a swimmer with a velocity of 1 mile an hour, aided by a current of the same velocity, is carried along the bank at 2 miles an hour, its validity is unshaken by Einstein's doubts. If, therefore, it

is possible to place light in the position of a swimmer or passenger, either the velocity of light is unique in principle (not merely in magnitude), or it does exemplify the law known as 'addition of velocities.' Einstein, as we know, rejects the second alternative, upholds the first, and maintains that it is consistent with the principle of relativity. I shall try, on the other hand, to justify the following propositions:—(1) That Einstein has failed to establish any analogy between the ray of light and a passenger; (2) that he has failed to see the point of the absence of such an analogy; (3) that, in one sense, the case of light is *not* unique, inasmuch as the velocity of *all* things is constant; (3) that the case of light *is* unique in this respect, that a special position is assigned to it in physics; (4) that a reason for this exceptional position, and perhaps even for the actual figure represented by c , can be given.

First, then, as regards the analogy, Fizeau's experiment will supply us with a valuable suggestion. Suppose a train like a hollow tube were constructed and set upon wheels, with no compartments or other obstacles in it, and with both its ends open. Suppose, further, that arrangements could be made (though it would be rather difficult) for a man to jump on a stool

between the rails, as soon as the train had passed, and thence to fire a bullet straight through the tube of the train, so that it should emerge at the other end. Would this bullet be a passenger in the train? Or would anyone contend that the speed of the bullet, as measured by the embankment, would be affected in any way by the speed of the train? Draw a simple diagram thus :



A is the point on the ground from which the bullet is fired, and *B*, perhaps 200 yards distant, is another point on the ground, towards which the bullet is aimed. It is calculated that a bullet's normal time from *A* to *B* is x seconds. Will anyone doubt for an instant that a bullet so fired from *A* would arrive at *B* in x seconds, regardless of the train's velocity? True, the extra draught through the train might slightly 'retard' it, but such retardation would be wholly irrelevant to the 'addition of velocities,' so far as concerns the bullet and the train. Now, obviously, the bullet differs from a human passenger, in that it does not develop a particular velocity by the aid of the train regarded as a body *at rest*. It does not walk

along the corridor with the same velocity that it would exhibit on the embankment or any other *terra firma*; and for that very reason, if the train should happen to be just 100 yards long, it would not pass through the train in $\frac{x}{2}$ seconds. For, as the train is moving in the same direction, it will have to go more than 100 yards before it is clear. A passenger, on the other hand, would walk his 100 yards in exactly the same time as he would walk it in a field, no matter what the speed of the train. Do not imagine, however, that the velocity of the bullet is modified by the forward progress of the train.

It travels always at the rate of 100 yards in $\frac{x}{2}$ seconds, but it takes more than $\frac{x}{2}$ seconds to clear the train, because it has to go more than 100 yards.

Now, in Fizeau's experiment there is a suspicious resemblance between the light passing through the liquid and the bullet passing through the hollow train. If the liquid can in any degree 'retard' the light, thus far it provides an analogy to the adverse current of air which delays the bullet; but neither in that fact nor in any other shall we find any analogy to the human passenger, or any excuse for

introducing the supposed relativity problem of the passenger and the train. Like the bullet, the ray of light develops no walking-exercise in which the train (here the liquid) serves as a body at rest. Were the light carried by the liquid, as a man by a train, we should have also to remark that the light, regarded as part of the train, must be carried along with exactly the speed of the liquid, like a box in the van. But is it not already manifest that the analogy is fast breaking down?

Before abandoning it, let us attempt, however, to place it in a more instructive aspect. Imagine now an ordinary train, with all its lights extinguished, going its way through an inky night. The darkness is not indispensable, but it may save us from external distractions. After a while a man goes to one end of the corridor, and starts switching an electric light on and off. At the other end of the corridor a mirror has been so arranged as to reflect back each successive flash to its source. Here the light, we may think at first, is more like a human passenger, because it has not, apparently, leaped in at the back, and because it travels up and down the corridor in a gentlemanly way. Or must we still reject the analogy on the ground that the light is not walking on the corridor regarded as a body at

rest? With what velocity, let us first ask, does it go to and fro? Inevitably with its own unwavering c . Does it move, then, with the same velocity relatively to the train as to the embankment? Yes, *but so does the human passenger*. The train is at rest in relation to the walking passenger when he walks on it at 3 miles an hour. He might just as well be walking on the embankment, which, for that matter, is itself a train when you consider it from the embankment of the sun. Thus far, then, the light resembles the passenger, but only when we realise that the walking of the passenger is an excellent display of constant velocity. Or again, you might send two boys into the corridor to toss a ball to and fro. In that case the ball would have two experiences. Whenever it was in a boy's hand, it would travel as part of the train, with the train's velocity and no other. Whenever it was in the air, it would travel, like the human passenger and the flashes of light, with the same constant velocity as if the boys were playing in a field. Or once more, if a return to the 'addition of velocities' be desired, the human passenger as he walks, the ball as it sails through the air, *and* the flashes of light as they dart to and fro, are simultaneously carried along by the train at, say, 60 miles an hour; and thus in its own

limited sense, the theorem of the 'addition of velocities' retains its validity. The bullet and the light traversing the liquid in Fizeau's tube are in a different position. Both take some fraction of time longer to clear their train than they would if it were at rest, but that fact, it is surely obvious, has nothing in the world to do with the 'relativity' problem of the passenger and the train. It is a fact analogous to the passing of a slower train by a faster, a little problem that will demand our attention before the close of this chapter.

Suppose next that a man tries to imitate the bullet, by jumping in at the back of a moving train, dashing through it and out at the other end. Well, he will have two ugly tumbles, the second of which will probably be fatal. To avoid this abrupt conclusion of the experiment, we will substitute for the train one of those moving staircases so dear to Londoners who travel by their own Fizeau-tube. The passenger advances over the level floor at the foot of the stairs with his constant business velocity of 4 miles an hour. Walking upstairs is something of an effort (one of the embankment sensations), and his rate will fall a little below his average; a fact analogous to the slight checking of the bullet as it meets a strong current of air. During his transit, however, the

staircase is hurrying the man up to the top of the stairs with its own private velocity; hence an observer on the embankment (*i.e.* anywhere not on the staircase) would certainly time him as passing over the distance from the bottom to the top—perhaps 30 yards—with a velocity greater than 4 miles an hour. This is as near as we can get to an analogy between a human passenger and the bullet or Fizeau's ray of light; and herein both the strong and the weak points are exposed. The analogy fails because there is nothing in the voyage of the light or the bullet analogous to the walking of the passenger on the motionless floor of the train or staircase. On the other hand, we are helped by our criticism to understand the real analogy between the velocity of light in the 'ether' (or *in vacuo*), the velocity of a ball or bullet as it flies through the air, and the velocity of a man walking on the surface of the earth, whether the surface be called 'the embankment' or 'the train.' In a word, the attempted analogy has brought us a step nearer to the thesis, anticipated ever since we first analysed the case of the swimmer in the stream, that the velocity of all things is constant. Human beings, however, are too wayward for easy association with the problems of science. The less we allow the intrusion of consciousness, the better it will

be for the clearness of the argument; for which reason it is an advantage to turn away from the variability of human effort to the bland uniformity of light.

As the meaning of 'constant velocity' is open to some elementary misunderstandings, perhaps it will be excusable to mention one or two of them here, with special reference to light. If a train running at 10 miles an hour, and another six times as rapid, were to start together from a terminus, and travel in the same direction, so that after an hour the faster was 50 miles ahead of the slower; and if at the end of the same hour a ray of light were despatched from the same terminus in pursuit of the trains, no one will deny, I imagine, that the slower train would be overtaken before the faster. And again, if the length of the two trains happens to be the same, no one will pretend that the ray will pass along the slower and the faster train in exactly the same time. By reason of the immense velocity of light, the difference will be less than measurable in our ordinary figures, but it must none the less exist. If not, the only possible inference would be that light did not move at all. Or again, we should have to maintain that the light from the remotest of the fixed stars took no longer to reach the earth than the light of a candle to reach the

corner of a room. In a word, light cannot be said to have velocity at all unless it takes time to cover distance, and therefore, more time to cover a greater distance. Thus far, then, there is no difference in principle between light and a train, or any other moving body. What, then, is its 'constant velocity'? It can mean nothing else but that, when light is considered in relation to its own 'medium,' and to nothing else (*in vacuo*, as they say), it always moves with its own velocity; and again, that this velocity is unaffected by any movement of the source of light. I use the word 'medium' without necessarily assuming that 'ether' exists, but as a reminder that the measurement of any velocity implies the existence of something analogous to the stream that does not swim or the road that does not walk. Now there is no objection to saying that the velocity of light is, in this sense, constant; but so, in the same sense, is the velocity of everything else. The swimmer swims his 2 miles an hour in the river, the walker walks his 3 miles an hour on the road, the bird flies its 30 miles an hour, or whatever it may be, in the air; and a hundred other parallel cases may be added. As to the movement of the source, there is not a scrap of difference between light and anything else. What is the analogy to the 'source' in the case

of the swimmer? Only two relevant things can be said to be in motion, the swimmer himself and the stream. To call the swimmer the source of his own velocity would (in this context) be rather absurd; so we must try what can be done with the stream. Now, a stream may advance with any velocity we choose to assign to it—2 miles an hour, or 10, or 50—and swimmers will thereby, from the bank point of view, be helped or swept along by it. But this, as by now we should understand, makes no difference to the swimmer's velocity. He continues at his own 2 miles an hour, and everything that he encounters or passes in the water, he approaches with precisely that velocity. The pace of the stream is wholly irrelevant. So likewise in the train, the passengers overtake or pass one another with their several velocities, and the velocity of the train itself has nothing to do with the question. So far as concerns their velocities, the train is *at rest*. Take away the body at rest, and 'velocity' means nothing at all, or nothing to the point. Or, if we catch ourselves drifting back to the 'addition of velocities,' it will be necessary to remember that a light (for example, a lighted lantern) can be carried along in the train, at the train's rate of speed, just as easily as a human passenger while he walks in the corridor of a train, or a

swimmer while he swims about in a bath on board one of our gigantic modern ships. Once get rid of the confusion between the stream and the bank, and it will be found remarkably difficult to discover a sense in which the velocity of light is constant, but other velocities are not.

There is also a simple piece of logic which cannot be neglected, though it appears to have received very little attention in connection with the doctrine of relativity. The velocity of light *in vacuo* is said to be constant in relation to all moving bodies, whatever their velocity. But if so, it must certainly follow that their velocities are constant relatively to it, and also to one another. Thus in any group of numbers, such as 24, 12, 8, 6, there is constancy of ratios; 24 is always twice 12, and 12 is always half 24: 8 stands always in a certain relation to 6, as does 6, reciprocally, to 8. Now, the world may not be composed of numbers in quite the simple fashion suggested by the genius of Pythagoras; but, unless some very fantastic meaning be given to 'constant,' it will need some rather violent evidence to convince us that the velocity of light can remain constant in relation to other velocities, while none of those others enjoys the reciprocal privilege. Mere logic, as its enemies love to call it, is rarely successful; nor do I pretend we ought

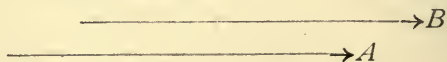
to be satisfied with anything less than a vigorous application of the logic to the case before us. I merely contend that, if the velocity of light can be made to escape from the dilemma, it will only be at the cost of arguing that light does not move at all.

In point of fact, the unique position ascribed to light in the doctrine of relativity does, I believe, depend to a large extent upon a confusion between two positions, which may be called the positions of the swimmer and of the stream. I will ask leave, therefore, to make here a digression, which yet is no digression, since in fact it bears very directly upon a large part of our argument. I invite the reader's careful attention to what may be called, perhaps, the first principle of motion. It is that, wherever any group of bodies, large or small, is in motion, their number must always be one less than the total number of bodies involved, the extra one being necessarily at rest. To this must be added that the minimum number of things required to make motion possible is three. Were there but one body, in the Parmenidean style, it could not be in motion; though neither, we must allow, could it be at rest. Were there but two bodies, these again could never be in the state of motion; for the position of A in relation to B could never be

altered without a corresponding change in the position of B in relation to A ; which is the same as to say that neither could move. But the moment a third body is created, the motion of any two of them becomes possible, so long as the third remains at rest. Which of them is taken as the point of rest it matters not, but one such point there must always be. For how can A 's position relatively to B be changed, unless the stability of C is assumed? Here, more than anywhere, the intrusion of consciousness must be forbidden; for a conscious being will either begin to think of itself as two, or else will forget that it is one. Three, then, is the minimum, but the same law must hold good of every group, larger or smaller; whence, if n be the total number of bodies in motion, the physical world must consist of $n + 1$. If all bodies were in motion, they would likewise all be at rest; or, to speak more accurately, neither motion nor rest could be. That all things are in a state of flux, and that motion is impossible, are two statements of an identical doctrine. Not *all* things, dear Heracleitus; say rather, all but one.

With a firm grasp of this first principle, we may now construct one or two useful illustrations, with a preference for such as prove motion impossible where only two bodies are

taken into account. Consider first the familiar phenomenon of a faster train overtaking a slower, when they are running on two parallel pairs of lines. You know, perhaps, that one is travelling at 60 miles an hour, the other at 30, and you say, therefore, that the quicker will pass the slower at the rate of 30 miles an hour. Should the length of the slower be exactly one furlong, you will calculate the time of the transit as 15 seconds, one-eighth of 2 minutes. Now, these velocities of 60 and 30 are evidently measured with reference to a solid and stable earth, whose intricate gyrations among the heavenly bodies are frankly ignored. So, too, when we speak of one train passing the other, it is assumed that both are going in the same direction, and this 'direction' is fixed by a motionless embankment, or something of the kind. So far, so good. No one, however, when it is fairly put to him, can well deny that, while *A* is passing *B*, *B* is simultaneously passing *A*. 'Passing' is a mutual operation, and the merest glance at a picture of the trains will assure us of the fact.



For while the engine of *A* is creeping at 30 miles an hour towards the engine of *B*, the tail of *B* is overtaking the tail of *A* with the same

velocity. Someone, perhaps, will begin to speak of 'direction,' and will object that the tail of *B* overtakes the tail of *A* only in so far as it is moving in the opposite direction. This will have the useful effect of reminding us that, when trains pass one another in opposite directions, their velocities must be added, in order to arrive at the rate of transit. We must be careful here not to be confused by the 60 miles an hour imputed to one of the trains, for that velocity is relative to the embankment, not to the other train. Each train is passing the other at 30 miles an hour, and their directions are opposite. They pass one another, therefore, at 60 miles an hour. But when trains behave in this ludicrous fashion (and several other perplexities might be mentioned), is it not almost time to bring them to rest?

The reader, I imagine, will condemn the whole account of the trains as sophistry! Yet it is not so much sophistry as a reminder of the impossibility of attaching any meaning to velocity and direction, if you once lose your hold on the fixed point of reference and try to think of only two bodies, *A* and *B*. Try the experiment more directly. Close your ears to the uneasy rustle of the universe, your eyes to the earth and sky. Banish all the vast plurality of beings, till nothing survives but just two

trains. With what velocity will they now pass one another? Whence are they journeying, and whither will they go? To strain the imagination to so high a pitch is difficult, but the effort will help us at least to understand that any conclusion affecting velocity must affect both space and time. If two bodies alone can never be in motion, neither can they have any existence in time or space. What is time when it ceases to be the 'number of motion'? And what is motion without distinction of points in space? But if only two points in space as yet exist, it is pardonable curiosity to ask where they are. Is one to the right or the left of the other, above or below. Or again, if only two things are as yet in time, what time is it, when nothing has happened already, and nothing is going to happen? On no account must an *interval* be allowed to insinuate itself; for an interval, whether measured in space or in time, will easily be translated into a third unit. The conclusion of the whole matter—by no means a barren conclusion—is that neither one thing alone, nor two can have 'existence.' The creative number is three.

After solemnly dedicating the triad to the shade of Pythagoras, let us return for a moment to our former trio, the embankment, the passenger and the train. The observer on

the embankment, we notice first, always assumes in his calculations that his own post of observation is motionless, while the train and the passenger execute various manœuvres. The passenger, again, is conscious of being at rest in his comfortable seat, and reflects, perhaps, that as the train is hurrying past the embankment, so must the embankment be hurrying past the train. He may, indeed, be slightly puzzled by the thought that his own stability is only that of the train, but perhaps he will satisfy himself by reflecting that, as part of the train, he is decidedly in motion. Thus far he has shown himself capable of grasping the embankment point of view as well as his own. But presently, when he goes for a stroll in the corridor, he will be puzzled anew. For now, he will argue, I can no longer be part of the train, since I am walking about on its surface. With this thought he passes, in fact, to the third point of view, which belongs by rights to the train. For the train is at rest in relation to the passenger who walks on it, and also to the embankment which goes scuttling by. Here again, then, we detect the same principle, that of any three bodies any two can be in motion, but only if the third is at rest. The problem of 'relativity' arises when two of the three stand-points are confused together, or when the existence of the third is forgotten.

Such, in fact, was the origin of the traditional error in the interpretation of the Michelson-Morley experiment. The authors of the experiment failed, in the first place, to understand that a swimmer moves this way and that in the water exactly as a passenger walks up, down or across a train. And further, they supposed themselves to be timing the rays of light from the bank, whereas really they were timing them as from the stream or the train. It was exactly as though a man on the bank had said to a swimmer, 'You must have been cheating; you have swum 50 yards across, and also up, the stream in exactly the same time as you took for the same distance when helped by the current.' To which the man would naturally have replied, 'My dear sir, you have been measuring my progress by points on the bank; but I wasn't swimming on that bank of yours; I was swimming quietly about in the motionless water, just as a man walks freely about a motionless train.' That this is the explanation of the Michelson-Morley affair I find it impossible to doubt. We are left, however, with the interesting question, how it was that they managed to arrive at a correct conclusion about the rays of light, when in the case of two ordinary swimmers they would certainly have made the usual mistake. Here, perhaps, we come a little nearer to perceiving the genuine

peculiarity in the position of light, which cannot be identified with the constancy of c .

But first it is expedient to get rid of a dangerous inaccuracy, which hitherto has deliberately been allowed to pass. More than once I have said that a moving body moves in relation to another moving body, and also to a body at rest. In point of fact, the relation of one moving body to another is not of the same order as its relation to the unmoving standard of reference. When the earth, for example, is regarded as being at rest, the vehicles that travel on its surface do not move faster or slower than the earth itself. The road does not compete in speed with walkers and carriages, but two men may rightly be said to walk at the same speed, or the carriages to go faster than the men. This is because the standard of spatial measurement, such as milestones, is fixed by the motionless earth, which measures also the velocity and the time. As soon as the earth itself is conceived to be in motion, the sun or some other body must have become the point of rest. When only three or four bodies are included in the problem, this need of a point of rest is easily apprehended. As the number grows larger and larger, it grows ever more difficult, until at last, when we deal with millions and billions, we slide imperceptibly into the hypothesis of a world wherein all

things are for ever in motion. Yet the principle holds good of the millions just as surely as of the original three. We must bear always in mind, therefore, that a body in motion is said to move relatively to a body at rest only because its movements are detected and measured by the fixed point of reference, not because there is any comparison of velocities between that which is said to be in motion and that which is now at rest. And further, we must brace ourselves to apply to the physical world as a whole this same irrefutable law, that where n is the number of bodies in motion, the total number of bodies is $n + 1$.

The mistakes of our forefathers are sometimes instructive. Often enough, when their conclusions were erroneous, their instincts and principles were right. Before Copernicus, in the long period when the bolder speculations of ancient Greeks were forgotten, the earth was taken as the immovable centre and standard, by which the direction and velocity of all things was determined. Round about the earth, away from it or towards it, a body might travel with greater or lesser velocity; but, once it became part and parcel of the earth, its travels were over. In those days, perhaps, some adventurous man of science might have proposed to treat rest itself as but one variety of motion. The earth, he might have argued, has

a zero velocity, and zero is one of the numerical series. Ingenious and stimulating as this proposal might have seemed to the curious, the effect of accepting it would have been to make motion impossible. The strongest point in the old geocentric theory was its retention of an immovable body or point. After the Copernican revolution, however, and when *e pur si muove* became the accredited motto of progress, the earth was robbed of its ancient stability, and all things were upside down. For a brief time the sun may have seemed to inherit earth's traditional privilege, but the tendency of thought was all towards the discovery of new motions. The sun itself was cut loose from its anchorage, and forced to join in the giddy quadrille. The art of measurement, too, was vastly extended, until light, as it sped across the celestial wildernesses, was timed like a runner in a race. The result, as we know, was to establish the constant velocity of light, upon which Einstein has based so large a part of his theory.

One thing, it seems to me, discovered unwittingly by Einstein and others is just this impossibility of motion unless there is a body at rest. Light is said to have a finite and constant velocity, to which no other bodies, waves, rays, or what not can attain. The actual figure of its kilometres is published, and all other

competing velocities are invited to envy and admire. But between a finite, invariable, unattainable velocity on the one side, and an immovable earth on the other, it is fair to ask what difference there is. Many differences, no doubt, for many purposes, but none whatever for the purpose of measuring comparative velocities. You cannot go faster than the track; you cannot go slower. But the relative speeds of the runners, whether looked at from the one point of view or from the other, will be the same in both cases. It would be a bold stroke for physical science to declare that light does not move at all; that it is not one of the swimmers, but the stream itself. The dull persistence of c would then take the place of the immobility of the earth. Amid all the crowd of jumping, flickering, oscillating bodies, light would stand calmly aloof as the permanent $n + 1$. But while the gain in practical convenience would be considerable, one must freely recognise that there is little chance of so great a revolution in the position of light. We seem to be too deeply committed to the comparison of its velocity with others; the visible evidence of its movement seems too overwhelming. Is it more overwhelming, I wonder, than the evidence of immobility when we stare at our mother earth?

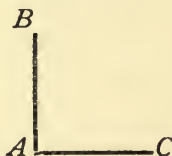
CHAPTER V.

THE UNIQUE POSITION OF LIGHT

TO REVIVE an old heresy, or to propagate a new one, is no part of my purpose. I am not contending for the immobility of light as an absolute fact, or absolute for physics; nor have I the least desire to evade the patent objection, that a velocity calculable at so many kilometres a second has a considerable claim to be regarded as real. The object of this chapter is to investigate further the position of light, to diminish, if possible, the danger of an unconscious confusion between the stream and the swimmer, and to criticise one or two doubtful assertions found in many accounts of the 'relativity' doctrine. Had the possible function of c as an unattainable velocity, not comparable to other velocities, but analogous rather to the zero of a motionless body, ever been carefully examined, a false uniqueness might not have been attributed to the mere fact of constant velocity, as though such constancy were peculiar to light. Assuredly there would have been less surprise at the ability of light to swim with the same velocity at every angle

to the stream, had it been clearly understood that the same talent belonged to every swimmer in the Seine or the Thames.

The error in the interpretation of the Michelson-Morley experiment arose, as was said before, from the belief that the earth upon which they and their apparatus were fixed was analogous to the bank of the stream. Not that it is necessary to make the error whenever you are, or suppose yourself to be, on the bank; but make it you certainly will until you have got rid of the 'retardation' theory by discovering the actual course of the swimmer. As it happened, however, on that famous occasion, the experimenters and their mirrors contrived to reflect the rays back along two different axes were all in the middle of the stream. The apparatus was like a pair of sticks, or part of a picture-frame, in the shape of a right-angle, thrown into a boundless sea.



It could matter nothing 'where' the apparatus was at any particular moment, for everywhere was the same. If a swimmer is placed at *A*, with two such points as *B* and *C* equidistant

from him, by no possibility can he take more time to swim from A to B than from A to C , or more time to return to A from B than from C ; provided, of course, that he is credited with a certain standard velocity in still water. His still-water velocity is, in fact, his only possible velocity; for in relation to the act of swimming all water is still. But why, it may be asked again, did Michelson, Morley and their friends get the correct result in the case of the two rays? If they were not on the bank, but in the water, how did they get there? It was not a question of getting there: the point is, they could not get *out*. There was no bank for their ark and its mirrors to rest on, unless they could sail beyond the borders of the physical world. There may or may not be a reason for distinguishing between light itself and the stream of 'ether' in which it travels, but no more is required to account for the case of Michelson and Morley than the fact that light is everywhere around and about us, no less than the water which surrounds on every side the swimmer whom, in an earlier chapter, we placed in the midst of a shoreless ocean. It is nothing to the point that the earth itself and other islands seem distinct from the river. The passenger in an ordinary train distinguishes the embankments and many other

objects from the train itself, and can easily imagine what his point of view would be if he were standing in a field and watching the train rush by. But this does not prevent him from discovering, or rather, from assuming that six feet across the width of the train measure neither more nor less than six feet uptrain or downtrain, and so can be traversed in the same time without change of velocity. Such was the situation of Michelson and Morley, although they knew it not. They supposed themselves to be on a bank looking for a problematical stream, while actually they were in the stream, and might have looked in vain for its banks. They accepted the swimmer's evidence because there was no other to confuse them. Yet they were puzzled, and went rather sorrowfully home, because prejudice had persuaded them, first, that they were on the embankment; secondly, that swimmers in general were retarded by currents. In Chapter II we noted that whenever the bank-observers, with their false presupposition, agreed with the passenger about any one of the three factors, space-distance, time-distance and velocity, they must always disagree about the other two. Had Michelson and Morley really been stationed on a bank outside the stream, they would certainly have fallen into dire confusion. As it was, the

refusal of the light-passenger to give false information to persons actually in the train (or stream) saved them from one kind of error; but, having failed to interpret the information correctly, they plunged headlong into another, and decided that the case of light was unique. Had they critically examined the behaviour of any passenger in any train, many inferences would never have been drawn.

The ground is less firm ahead of us when we ask more explicitly whether light does indeed travel in a medium, such as the 'ether' of modern science, or even the 'vacuum' which the ancients alternately affirmed and denied; or again, whether it may not in the end be the simplest hypothesis to regard light itself as but one manifestation of the 'substance' which permeates the whole physical world, a manifestation possible in circumstances which it would be partly for physics, partly for physiology, to explain. To offer here any definite opinion would be an impertinence, but the mere presentation of the hypotheses is excused by its bearing upon the further question, strictly relevant to our discussion, whether measuring of the velocity imputed to light may not be a process somewhat analogous to the act of a passenger who should attempt to determine the velocity of his train

by first marking out a distance in it, and then solemnly timing himself as he walked to and fro. Light is not a train in which we journey, but it is the physical medium or means of vision. Is it possible, then, that, when we succeed in assigning to light a finite velocity of so many kilometres a second, we are only expressing by an indirect method the limitations of our own power of vision? This is no sudden excursion into idealism or any other kind of metaphysics. The kind of limitation I mean would be similar to those expressed by such terms as *minimum visibile* or *minimum audibile*. If there must come a point for all human beings—despite variation round the point of normality—when an object is too small to be seen; and an analogous point when (to speak paradoxically) a sound is too faint to be heard; may there, or must there, not also be a degree of velocity which betrays to us no evidence of itself in the form of visible signals? It is useless to object that the velocity of light was determined by calculations, not by simple observation; for the calculations cannot well have been based upon anything else but visible phenomena. The premisses themselves would thus be vitiated long before the arrival at any conclusion.

In the mere notion of seeing things quickly

or slowly there is nothing outrageous. Every cricketer knows, or at least firmly believes, that some batsmen see the ball quicker than others; and some time ago, if I remember rightly, a kind of psychological machine was invented for registering the speed with which different persons become aware of a luminous spot. We need not even give rein to conjecture about signals wholly beyond the human power of vision. When distances are so stupendous as those which divide the stars from the earth, or one star from another, delay in perceiving the arrival of a signal might lead to the most astonishing miscalculation. We might go out to receive a letter when the postman had been knocking for a year on the door. Such a trifling misfortune would in no way disturb the symmetry of our calculations. Our periods of time, and our consequent estimates of distance and velocity, would all be founded on information conveyed by the same postman; and when the postman alone, by his daily or annual round, brings the time to the district, he can never be early or late. Very easily, therefore, might we construct a system of distances and velocities, which to light itself, could it offer a criticism, might well seem inexplicable and absurd.

As psychological, though not properly meta-

physical, this speculation is, I allow, rather doubtfully congruous with the general character of the enquiry. Nothing vital to my argument is contained in it, and on a later page I shall venture to offer another, more definitely physical, suggestion concerning the actual significance of c . It seemed worth while, however, to adopt this means of arousing a little scepticism about the definite figure, in case the mere sound of 300,000 kilometres a second should prevent consideration of the analogy between the status of light in modern physics and the status of earth in the obsolete natural philosophy. The genuineness of that analogy will not so easily be disputed, and this is the reason for my previous statement that light, though usually ranked as a swimmer, may also be called upon to play the part of the stream. No swimmer moves faster or slower than the stream; for the stream cannot move in relation to him, until the bank makes its irrelevant intrusion. The stability of the earth was once constant in relation to all velocities, because it measured them all by its own standard. If the analogous position has not yet been ascribed to light, may not this be through failure to grasp the first principle, that behind and beneath the vast turmoil of movements lies the motionless $n + 1$?

That principle does not, however, oblige us to declare any preference for one particular body out of a group. On the contrary, such preference would often be misleading and sometimes impossible. May we not, then, allow light to continue its rapid career, and select something else as a general point of reference for the measurement and comparison of velocities? Why not pick out an omnibus, for example, the first you chance to meet in Trafalgar Square? True, it would be inconvenient if the omnibus happened to have gone to Brixton just when you needed its help for estimating the velocities of trippers on Hampstead Heath; but practical objections should not be allowed to confuse the issue when a scientific principle is at stake. Theoretically, indeed, there would be no objection to the omnibus, unless it were found in its complication with that human instability of behaviour which always agrees so badly with mechanical conceptions. It is the human swimmer, with his varied aims and emotions, who has blinded us to the nature of swimming, and doubtless we shall be well advised to avoid all standards of reference that may involve us in the same kind of complexities. Yet nothing can cause so grave a misconception of the problem as the belief that light is unique in the possession of

a constant velocity. Those innocent little words, *in vacuo*, by which we qualify the assertion of constancy, have served to hide the truth they might well have revealed. The true intention of *in vacuo* is to deprecate irrelevance. We might profitably make use of the same phrase in connection with every velocity that we desire to examine. The velocity of every swimmer is constant *in vacuo*; in other words, it is constant if you attend only to the water, the medium in which he swims, and harden your heart against the protests of the bank. Were the world composed of nothing but water and swimmers, so that our minds were free from all notions of velocity and direction derived from a world outside the water, the stream or ocean would naturally be taken as 'at rest,' and it is a question how far differences in velocity between one swimmer and another could ever be remarked. Yet, probably, some system of points and directions would be invented; little groups of swimmers, in which one was taken (unconsciously perhaps) as the fixed point of reference, would attract peculiar attention, and the affair might end in the discovery of some superlative swimmer whose velocity was unsurpassable, finite and constant *in vacuo*. In our own world, with its medley of land, water, sky and other things,

water and swimmers are merely one phenomenon among many. Banks and other standards of reference are inevitably adopted; we do not speak of swimming *in vacuo*, nor does it readily occur to us to think of constant velocity, as we stand on the banks of a stream and watch boats, swimmers, sticks, leaves, and bits of paper passing by. The comparative ease with which the constancy of c has been recognised cannot, as we have seen, be attributed to the uniqueness of the fact, nor yet to any very clear thinking on the subject. It is partly due (as I am disposed to believe) to the instinct for securing an invariable point of reference, but more obviously to the virtual omnipresence of light, and to the relatively trifling effort of abstraction implied here in those two words, *in vacuo*. Light is everywhere round about us; it does not flow between visible banks, on which you can measure irrelevant distances and directions; only some sinister descendant of Berkeley would pretend that it was absent even in darkness! Its velocity, too, if we insist on thrusting it into the competition, surpasses all others, if only because (as I cannot help suspecting) the project of discovering with the aid of vision a velocity greater than light's far out-Quixotes the Don himself.

The velocity of light, then, enjoys a unique

position, but is not entitled to it merely because its constancy is unique. In many ways, too, the importance of light has been grossly exaggerated. Again and again, for example, it is asserted by Einstein and others that only by light-signals can we determine the simultaneity of events. This, surely, is a preposterous fiction. How do we know, for instance, that the striking of a clock on the mantelpiece is simultaneous with a knocking on the door? Or how do we judge that the perception of a certain odour is contemporary with the arrival of a pig? It is important, too, to remember that only by the same criterion can we judge that events *are* simultaneous, and again, that they are *not*. By what kind of light-signal *alone*, pray, do we judge that the noise of a gun does not reach us simultaneously with the flash or the appearance of the smoke? Still more pertinent is it to analyse our judgment of simultaneity in cases when we flatly reject 'the evidence of the senses.' When, for example, a sound and a flash do happen to strike us as simultaneous, why do we promptly decide that 'really' they were not? Evidently because judgment is a function of reason, not of one sense, or two, or five. Sight, it is true, has a wider range than any other sense, but that has nothing to do with the question of judgment.

In this matter, one may say without discourtesy, the physicist does not speak with peculiar authority.

With some amazement, too, do we read the passage in Einstein's book where he argues from the case of a train moving along an embankment (§ 9) that simultaneity is relative. Two points, A and B , are marked on the embankment, and M is the middle point between them, as determined by the application of a measuring-rod to the embankment. Rays of light are despatched from A and B in the direction of M , so that a man standing at M judges their arrival to be simultaneous. Meanwhile a train is passing in the direction from B to A , and therein sits a passenger meditating on time and space. The train stretches all the way from A to B (and as much farther as is necessary), so that every event on the line AB has a corresponding position in the train. 'Just when' (as Einstein says with rather dubious logic) the flashes occur at A and B , a point M' in the train corresponds to M on the embankment. At M' sits the thoughtful passenger, but because the train is moving all the while in the direction of A , he is bound to see one ray before the other; for already, by some fraction of a millimetre, he must be nearer to A than to B . But what on earth is this supposed to

demonstrate, unless that a passenger who proceeded to infer that the rays did meet at M' , or did not meet at M (*i.e.* did not leave A and B simultaneously) would be better fitted for almost any occupation than for science? If the man knows the alphabet of reasoning, he will no more make such an inference than he will swear that the sound of thunder reaching him two or three seconds after his perception of lightning can have had nothing to do with the flash. He knows that, when M' in the train has passed M on the embankment, he cannot get back to the line drawn at right angles from M through the train, unless his own velocity on the motionless corridor be greater than the train's velocity on the motionless earth. Should it happen that the train was moving at no more than three or four miles an hour, he could, if he pleased, remain at the original M' , merely by walking with the required velocity 'against the stream,' until his face clashed with the back of the train. But since most trains move rather quickly, or since, by hypothesis, he is remaining at rest in his seat, he will have sense enough to know, should the question occur to him, that he cannot say whether the rays met at M or not, or, in other words, whether the flashes at A and B were simultaneous. The huge velocity of light, when

compared with such loiterers as trains, might cause him, indeed, to make an error of observation, but that again would prove nothing at all. Let us try something a little simpler. The inordinate velocity of light is sometimes rather tiresome, and trains make too much noise.

On the same side of the street, in houses *A* and *B* some forty yards apart, live two comfortable citizens, whose places of business oblige each to pass the other's door every morning. On the pavement between the two houses draw a line of Euclidean straightness from *A* to *B*. Bisect it and mark the middle point as *M*. In the road, some few inches away, draw another line, parallel to the first, and equal in length. Bisect it at *M'* and join *MM'*. Fetch the nearest policeman to act as observer, and place him at *M'* with strict orders to note whether the two gentlemen meet exactly at *M*. At the first stroke of ten by a clock equidistant from both houses, each of the householders appears on the pavement and begins to walk towards the other with a uniform velocity of 2 miles an hour. Almost at the last minute, unfortunately, the constable spies a shilling in the gutter, and moves a pace towards *A* to pick it up. Too late is it now to regain his deserted post; one citizen passes

him before the meeting with the other occurs. Will the constable, then, proceed to argue that simultaneity is relative, or that the exits from the two houses cannot have been simultaneous? If so, his pay should be stopped. As an honest and sensible man, he will frankly allow that, as he was not himself at M' , he cannot say where the collision occurred. To present the same point is another way; if the observer had been stationed at C , a point somewhere between A and M , and had seen the walkers meet precisely where he stood, would he then, knowing their velocities to be identical, have argued that they must have left their doors simultaneously? Such a question deserves no answer; and yet the sole difference between the two citizens and Einstein's two rays of light is the superior velocity of the rays. No, the real problem of simultaneity has nothing to do with the dogma of light-signals. A good example of it can be constructed with the help of the diagram in Chapter II. For, in favourable circumstances, the observer at A will *see* the passenger walking from C to D , though actually his course is CD' . The observer, therefore, will regard the arrival of the passenger at D as simultaneous with a certain position of the hands of his watch. But D (as distinct from D') is by this time some yards

away from the passenger, who never, in fact, has been anywhere near it. What, then, is simultaneous with what?

These comments on simultaneity encourage us, lastly, to speculate on the possible scientific condition of a world peopled entirely by the blind. The range of the other senses, feeling, taste and touch being evidently narrower, we may assume that hearing would come to occupy the position assigned in our own world to vision. Now, since direct apprehension of distant events would be limited to the range of hearing, means of measuring distance by sound would be imperatively required for scientific purposes, and to this end great pains would be spent on measuring the velocity of sound. By various researches it would presently be demonstrated that sound travelled *in vacuo* with a constant velocity, that this velocity was finite and (for a world of sightless men) unattainable by anything else. Much importance, finally, would be attributed to a crucial experiment, in which a gun was fired from a point equidistant from two lofty screens, so arranged in relation to the source of the sound that two echoes must return along different axes, at right angles to one another. To the amazement of all the listeners, the echoes would run a dead heat. Comparing this result

with the notorious fact that more time is required for a swimmer to cross a stream and return than to swim the same distance half upstream and half down, the men of science would be enabled to make some momentous deductions. The only weak spot in the deductions would be that between the case of the echoes and the case of the ordinary swimmer there was, in fact, no difference at all.

CHAPTER VI

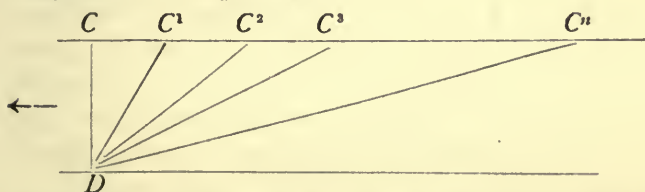
EUCLID, VELOCITY AND DIRECTION

AT THE END of the second chapter I used, with some hesitation, the term 'relativity of direction.' Whether or no that use of 'relativity' be defensible, it is necessary to assert here, even more explicitly than hitherto, that the 'relativity' lately come into fashion finds its principal residence in the brains of some eminent men of science. The supreme importance of direction has been almost ignored, and the very form of the problem, as so many have stated it, depends upon a misconception of its nature. 'Retardation' and the equivocal 'addition of velocities' have distorted the whole enquiry. For when the swimmer is judged to require more seconds to swim a given distance in one direction than in another, the sole reason for the variation in his time-table is the varying number of yards that he swims. Moreover, the yards and seconds used by the swimmer are (in every relevant sense) precisely

the same yards and seconds as are employed for the measurements on the bank. The same applies to the passenger in the train, and never, I imagine, would the fact have been doubted, had the lesson of the analogy between the stream and the train been clearly apprehended. If the passenger whose case was illustrated by the diagram in Chapter II did *not* walk 5 embankment yards in one embankment second, he could never appear to the observers to walk along the line CD , nor could he ever arrive at their D on the stroke of his scheduled time. Suppose the line CD had been carefully chalked, and the passenger had proceeded to walk along it, the soles of his boots would then have been covered with chalk, but the spectators would have vowed he never had walked along CD , and the chalk they would have scouted as a fake. Here, as in every analogous example, there are no elastic yards and seconds, no Fitzgerald contractions, no shrinking rods or Jack-and-the-beanstalk cigars, no Gulliveresque transformations of stature. These are purely gratuitous fictions, invented to palliate the ignorance of one simple fact, that the swimmer (or walker) does *not* travel along the measured line which his critics expect him to follow. He swims with his usual velocity in the river: they talk as though

he were swimming on the bank. So again, when we examine the questions belonging to 'addition of velocities,' the 'relativity' is found to be mythical, as soon as we discriminate between the several velocities, set each in its proper relations, and perceive, in a word, that none of them are added; though we can, if we please, make up a composite velocity and attribute it, somewhat irrationally, to one of the contributors alone.

Nevertheless, the real fact to which, in a sense, the error was due, demands our renewed attention. Repeating here a portion of the diagram in Chapter II,

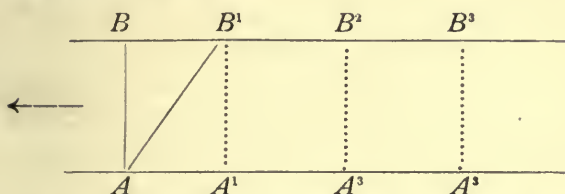


we can easily see how the embankment error might progress in the manner indicated by the lines C' to C'' , where the growth of the hypotenuse symbolises the ever-widening deviation from fact. To refer again to our previous figures, the observers might, in the case of a much faster train, have allowed one minute, instead of one second, for the passage from D to C . The passenger would then have been

obliged to walk along a hypotenuse of 300 yards, in order to reach the observers' C at the time prophesied for his arrival. Magnify these distances to astronomical proportions, and it will be found that a ray of light supposed to be travelling a distance of, say, 92 million miles, from D to C , in so many minutes will have actually to travel (on DC') 115 millions, to save the earthly sages from disappointment.

The problem, it is now essential to understand, does not require the presence of anything analogous in every detail to the behaviour of the passenger in the train. Neglecting, for the present, the promenades in the corridor, let us advance to a new presentation of the more important facts. If a man is sitting on a railway embankment, and a friend in a passing train desires to hit him with an orange, at what precise moment must he throw? Should one ask, perhaps, how *hard* he is going to throw, the form of the question must be disallowed. For though it is well to remember that the orange need not be thrown any harder from one point than from another, the notion of force suggested by 'harder' and 'softer' we cannot admit. It is enough that the orange will have a certain velocity, no matter what, and that this will be analogous to the still-water velocity of the swimmer. In the appended diagram A is

the point on the bank, while the B line shows the path of the train.



BA is the shortest possible line from the passenger to the target, as measured by Euclid's rules. Now, the right way to hit A is, clearly, not to aim at it, as though down a slanting line like B^1A , but to aim down a Euclidean straight line at right angles to the train, such as B^1A^1 or B^2A^2 . The orange will travel (while we keep to Euclid's terminology) along a certain hypotenuse, selected to suit the pace of the train and the distance from train to bank; but, if the passenger *tries* to throw the orange down the proper line, it certainly will miss the mark, and may almost re-enter the train by a window nearer the engine; not because of the wind, as we might carelessly allow ourselves to imagine, but because of the velocity of the train. Wind and gravitation we must ignore; we cannot here take account of more than two velocities, and the intrusion of a third would, perhaps, obscure important points of principle.

The most interesting facts connected with

this experiment are (1) that the man at A will (almost certainly) see the orange coming along the line BA ; (2) that BA is in fact the only line by which the orange cannot possibly travel, *unless the train is at rest*. For, manifestly, if the orange is released at any point before B , its line will not be BA ; while, if it gets loose exactly at B , it will necessarily miss, not indeed an ordinary dimensional man, but a man who can sit on the geometrical point A . The Euclidean BA , that is to say, is the 'straightest' and 'shortest possible' line to A when the motion of both B and A is prohibited; but, whenever the train is in motion, the line from B^1 , B^2 , or whichever it may be, will still be the 'straightest' line to the target, but by no possibility can it ever coincide with the Euclidean BA .

How well Euclid knew his business! As we are now reaching the limits of his geometry (or in truth have already transgressed them), a word in defence of his reputation will not be out of place. The whole meaning and truth of his geometry are exhibited in this vital fact, that BA is the shortest and straightest line only when the two points are at rest. It was not for Euclid himself, in a technical handbook, to discuss the presuppositions of his science; but the philosophers of that epoch, none more clearly

than Aristotle, regarded the whole of pure mathematics as outside the world of motion and time. When Aristotle calls the facts studied by arithmetic and geometry 'eternal,' what he means is that they stand in no relation to time, and therefore in none to motion. Had an impetuous pupil rushed in one day, exclaiming at the top of his voice, 'In the *real* world, good master, there are no geometrical straight lines,' the philosopher would have marvelled at nothing save the incurable taste of youth for the obvious. He might, however, have thought fit to ask the young gentleman whether the whole of 'reality' was comprised in the world of motion and time.

While speaking of Euclid, we may seize the opportunity for saying also a word about 'dimensions.' The fashion of calling time a 'fourth dimension' is a fallacious and tiresome mystification. Whoever imagines himself to be thus correcting the wisdom of Euclid has gone sadly astray. Ancient geometry recognises only three dimensions (often only *two*), because it deliberately excludes time and motion. And since the meaning of the *three* depends wholly on that exclusion, to call time a *fourth* dimension is to talk solemn nonsense in the most approved Delphic style. A man who can add time to Euclid's three dimensions

should be capable of catching a rainbow and penning it with three of his sheep. On the other hand, when you make a different abstraction, and pass into the physical world, where motions or 'events' take the place of points in a motionless plane, the three traditional dimensions coalesce into 'direction,' and the only other dimension is 'velocity,' wherein distance in space and distance in time are fused into one. True, you cannot locate an event without something to start from, but neither can you make a Euclidean construction until you have selected a point of origin, which for some reasons it may be convenient to regard as any one of the three angular points of a triangle. The geometry of motion requires a group of three moving, or rather moveable bodies, any one of which may be selected as the fixed point of reference for the location in time and space of the other two. Whether 'dimensions' is a suitable word for this geometry may be questioned, but at least let us abandon the unmeaning attempt to thrust time as a fourth dimension into the essentially timeless kingdom of Euclid.

After this brief parenthesis we must return to the orange. Thus far there is nothing mysterious in its behaviour. It merely imitates our passenger who was asked to go along

a line at right angles to the embankment, from C to D , so as to arrive at a particular moment. This he could only do by slanting across the carriage from C to D' , carefully avoiding the actual CD . But since in the throwing of the orange we may have appeared to get rid of some of the features belonging to the passenger in the train, we must now observe that no change of principle has been introduced; for the flight of the orange is analogous in all essential respects to the course of the swimmer in a flowing stream, or to the walks of the passenger in the train. In the first place, the time of the orange's journey, whether it be thrown from B^1 , B^2 , or any other point, will always be the same. This is involved in the selection of a particular point, to suit the speed of the train. Less obvious, but equally true is it that the orange preserves its uniform velocity, no less than the swimmer in the stream. If we endow it, for instance, with a velocity of 10 yards a second in 'still water,' that is to say, when thrown from a motionless B to a motionless A ; and if the distance from B to A is fixed at 10 yards; the orange, no matter where it has to be despatched on its journey to A , will travel, with its constant velocity, exactly 10 yards in one second. At the critical moment it goes to the train-bank, and takes a header into the

stream. For 10 yards or 1 second it rolls across the current, and is pulled up sharply at *A*. Were it taunted with having swum, say, 30 yards at three times its reputed velocity, its answer, correct and irrefutable, would be that the odd 20 yards and the extra velocity were an impertinent contribution by the stream. What is more, if the orange were a sophist, it could probably get the watcher at *A* to swear that *BA* was the actual line of its flight, a distance of exactly 10 yards. For the witness would be deceived by the optical *fact*, which some, no doubt, would call the optical *illusion*. To trade on errors of observation is, however, not the purpose of the argument. The analogy to the swimmer is genuine, and is made all the more valuable by the omission of a visible stream, and by the fact that one of the banks is running along with a speed of, perhaps, 60 miles an hour. Even the thrower of the orange would be involved in the confusion, unless he actually *knew* that he must hurl his missile before arriving at *B*. For he, if he looks at the right moment, will see the orange strike the mark, and, not allowing for the velocity of the train, may easily suppose that he has thrown it 'straight' at *A*. Thus he and his friend on the bank will be remarkably like two observers watching a swimmer go

'straight across' a stream, and timing him as though he had travelled along the straight line measured by themselves.

But now, as was recently hinted, we must take the momentous step of abandoning Euclid. For the sake of simplicity, I have described the path of the orange as a hypotenuse, and have thus represented it in the diagram. But this, of course, is a misrepresentation. The same reason that makes it impossible to throw the orange along the line $B\bar{A}$ makes it impossible for it to travel from B^1 or B^2 to A along a Euclidean straight line. The actual path of the orange will always be what we call a *curve*. And what is a curve? In process of answering this question we cannot fail to throw fresh light upon the problems treated already in previous chapters, but treated on too narrow a basis. The geometry of motion is the geometry of curves. It contains always two, and only two, elements or dimensions, velocity and direction. One of its advantages over the Euclidean geometry of rest and straight lines lies in not being hampered by the opinion that it is impossible to be in two places at once. In Euclid's geometry it is impossible, not only to be in two places at once, but even to go from one place or point to another. Nothing *happens*. When Euclid speaks of drawing a

line from A to B , nothing moves from one point to the other; he is merely reconstructing a figure already analysed, so as to show its connection with other propositions or postulates of his geometry. In the geometry of motion there are no 'places' or 'points,' except in so far as (by the first principle of motion) some fixed point of reference must always be selected as $n + 1$. Thus, when a body is regarded as actually in motion, it is *nowhere*. Zeno, I suspect, was well aware of this when he amused himself with constructing his famous puzzles; but by pretending that continuous motion must be made up of discrete points or moments, he was able to prove the impossibility of motion. It is impossible, if you try to build it of moments, each of which is essentially at rest. A moving body, however, is never anywhere at all; for which reason, perhaps, it finds no difficulty in travelling two distances and times simultaneously. 'Simultaneously' is the best word at our disposal, though the bias of language is too Euclidean for perfect clearness. Its meaning in the present context cannot be better explained than by re-examining the flight of the orange.

When the orange passes from the hand of the thrower to the target in a curve, what does this 'curve' represent? It represents two

times and two distances, or, in other words, the velocity of the train and the velocity assumed by the orange as it leaves the thrower's hand. Suppose the velocity of the orange to be 10 yards a second, and the velocity of the train three times as great. Then, in the second following upon the projection of the orange, the train goes 30 yards in its own line of direction, while the orange goes 10 yards at right angles to that line. What, then, is the length of the curve described by the orange? It is impossible to doubt that its length is $30 + 10$. The orange travels both distances and both times, and at first we seem to remark a curious distinction. For we should certainly say that only *one* second was involved in the whole proceeding, whereas in adding the distances, we add them, so to speak, end to end, and make a line $30 + 10$ yards in length. Yet the difference is not fundamental. There is a time-curve and a space-curve; the one unifies two periods of time, the other two spatial distances and directions. Better still, perhaps, is it to say that there is a single velocity-curve which embraces the whole process; but in any case the first step is to free the mind from the commercial dogma, that one and one make two.

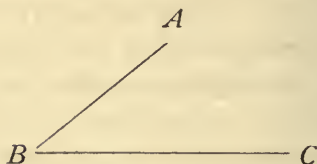
Meanwhile, the reader may be wondering what evidence can be offered in support of the

assertion that the length of the curve is the sum of the two distances travelled, respectively, by the train and the orange. The evidence is clear enough, if we return to that most useful example of the passenger and the train. When discussing the 'addition of velocities' we examined chiefly the case when the train travelled so many miles on the line, and the passenger so many yards in the corridor, walking in the direction of the engine; and at the end of the hour we added both distances together, though both had been travelled within the same hour. Now the total distance travelled, in such a case, by the train and the man together is, properly speaking, a curve exactly analogous to the curve described by the orange. At first we may be inclined to question this, because, when the man walks in the same direction as the train, we seem to add the two distances together in a straight line. The correct answer to this, no doubt, is that in the geometry of motion the straight line does not exist, unless you can succeed in regarding it as a species of curve; but as this may not be a suitable moment for discussing that question, it will be simpler to exhibit the analogy to the orange by supposing the passenger not to walk towards the engine, nor yet to construct the still more difficult curve involved in walk-

ing directly towards the tail of the train, but to content himself with walking across his carriage, from right to left or from left to right, so as to transfer himself to the side of the carriage where he expects to find the platform when he arrives. He walks, let us say, five yards in one second, at right angles to the direction of the train, and during the same second the train advances 15 yards in its own line. The length of the curve is then $15 + 5$ yards, and cannot possibly be anything else. It is merely like bending a supple stick into a new shape, or perhaps a piece of string would give a clearer illustration.

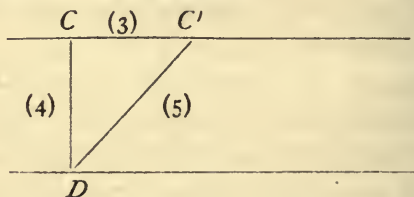
By considering the nature of a curve we thus arrive at an interesting point of difference between the two geometries. For in the geometry of rest we are accustomed to learn (*Euclid* I, 47) that, in a right-angled triangle, the *square* on the hypotenuse is equal to the sum of the *squares* on the other two sides; while in the geometry of motion, which knows nothing of straight lines and squares, the hypotenuse is replaced by a curve, and this curve-hypotenuse, if we may so name it for a moment, is equal in length to the sum of the other two sides. We thus get a kind of key to the transformation of the one geometry into the other. For, whenever the Euclidean straight lines

make an angle (not necessarily a right angle) thus :



we can always (with data analogous to those which we have about the train and the orange) replace the lines AB , BC by a curve equal in length to their sum. We might call this the velocity-curve.

With this new fact at our disposal, we are obliged to re-examine the behaviour of our original passenger who walked about on the diagram of Chapter II. A portion of it may conveniently be repeated.



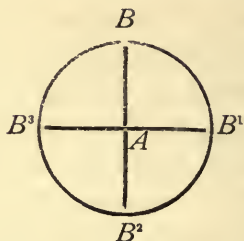
When refuting the 'retardation' theory, we proved indubitably that the passenger walked 5 yards along the line DC' (just the same class of yards as he would have walked on the embankment); but these 5 yards and this straight line DC' exist only in the Euclidean world of

the train. For the train is at rest in relation to the walking, and so one walks in it (so far as Euclid allows of walking) on Euclidean principles. At the same time, however, the train itself was running Euclidwise on the motionless earth at the assigned rate of 3 yards per second. When, therefore, we look at the whole performance from outside, taking the embankment or some other piece of ground to be at rest, and regarding both train and passenger as in motion, we see the passenger describing a curve, and know that the length of the curve must be neither 5 yards nor 4, but $4 + 3$. For one exciting moment the 'retardationists' will believe that they were right after all. For was not 4 yards the distance they allowed him in one second, when the train was to travel 3 yards in the same time? But no, the 'retardation' error is really exposed more patently than before. The man's curve is 7 yards in length merely because they *assigned* him a velocity of 4 yards per second, and he did his best to oblige them. As he was actually walking under Euclidean conditions, his method was to walk 5 yards instead of the allotted 4; but they assigned him these 4 yards per second on the assumption that he was *not* walking under Euclidean conditions, *i.e.* on a body at rest. And further, they sup-

posed him to traverse the Euclidean line from D to C , 4 yards in length, while in point of fact (a) he walked 5 yards along the Euclidean line DC' , (b) in the non-Euclidean world of motion he described a curve of 7 yards, merely to oblige them. Left to his own devices, he might have walked the 4 yards across the train in $\frac{4}{5}$ of a second; during which time the train would have moved $2\frac{2}{5}$ yards ($\frac{4}{5}$ of 3 yards); so that the length of his curve would have been $6\frac{2}{5}$ yards. Or again, he might (if the train were wide enough) have walked 5 yards across it in one second; the train would have gone 3 yards in the same time, and the length of the curve would have been 8 yards. Here, then, we do indeed seem to have stumbled into 'relativity.' For this double performance of the passenger's in the Euclidean and the non-Euclidean worlds seems to confound all estimates of distances and times. What is more, we have considered only two out of the vast complex of velocities and directions really involved. It staggers one to think of the others. No catalogue of them even begins to be possible. Let us be content for the moment to imagine the effect of introducing no more than two others, the rotation of the earth and its orbit round the sun. Where now is this unfortunate man? In what direction is he going?

What is his exact velocity? How shall we map the curve of his journey? And why, most strange of all the facts, does he display no evident symptoms of *mal de mer*? It only shows how deeply our emotions depend upon our knowledge. Nevertheless, we are not bound to assume that each additional velocity and direction included in the totality must, as it were, cancel those hitherto comprehended. On the contrary, each one preserves its constancy when viewed in its proper relations. Just as our passenger does quietly walk his five yards along a straight line drawn from one point to another within the train, so again, when we regard both him and the train as in motion, does he describe the given curve; and so, as each new velocity and direction is taken into account, will the individuality of every element in the growing complex be preserved.

While we were considering the problem of throwing the orange in relation to Euclidean parallel straight lines, we found that one line of approach to A , namely, the straight line from B , could never be used. Let us next try the effect of introducing the circle, and with it rotation. Take the same line AB and treat it as the radius of a circle with A as the centre.



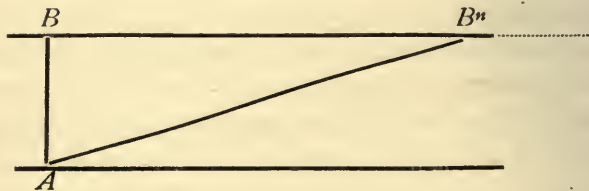
This circle we will regard as a merry-go-round, on which is riding a great host of cockneys, hurling oranges at A from every point of the orbit, each one of them aiming along one of the radii which converge upon the centre. The curious thing is they never can hit it. For since all the radii are equal, and each one is the shortest distance to the centre, every orange must miss the target for the same reason which prevented the passenger in the train from using the line BA for his attack. Naturally, I am not asserting that it is impossible so to hurl a missile from the rim of a whirligig as to strike the centre. The point is that the successful missile can never travel along a Euclidean radius. So long as the oranges are aimed 'straight' at him, the owner of A can sit on his housetop and laugh at his enemies.

Were such a performance to be organised, what would be the result? In the world of plane superficies, the shape of the orange itself would be out of order. For the disc-like

circle, therefore, let us substitute the impressive rotundity of the sphere. If from every point on the surface of a sphere the same perpetual shower of missiles were aimed at the centre, each one of them would be like a walker on a tight-rope striving to cross the sphere on a diameter, and each would be astonished to find that his own rope did not pass through the centre. Moreover, since the oranges would be raining perpetually from every point on the surface of the sphere, without gap or intermission, a not less perpetual series of collisions would knock each equilibrist off his rope. The result would be (unless we cling to the hope of a perfect sphere with a hollow at its centre) the formation of a solid mass with a spherical tendency never perfectly fulfilled. A lack of balance, a certain awkward lop-sidedness, would faintly mar its symmetry from the first. Should it ever be used as a billiard ball, the players would always be doubting the sincerity of its roll. Thus may we dimly guess, if we venture into cosmogony, how the earth or any similar body could, in a purely physical sense, be created. Within a vast revolving sphere, an endless hurricane of particles (to choose an untechnical word) might rage round an intangible centre, or perhaps an indefinite number of centres, and so build up eccentric globular

masses. The defects of the picture are obvious enough, but at present it is not worth while to discuss them, since our whole hypothesis, so far, is too Euclidean. We must get rid of the straight-line radii, which have no real place in the geometry of motion, and with them must go the whole Euclidean circle, circumference and all. In Euclid's circle radii and circumference stand and fall together. The rotating sphere (if that be the right name for it) obliges us to think only of curves; and what this means is that the 'centre' of a system of curves could never be the centre of a Euclidean circle, or of a sphere regarded as being at rest.

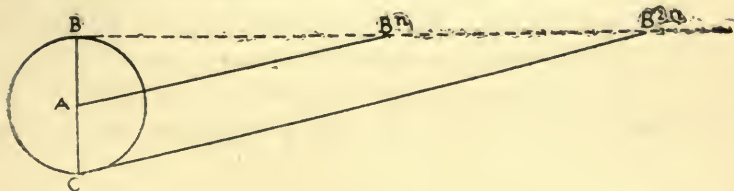
Nevertheless, it will be useful to return for a moment to the train and the parallel lines, for the purpose of advancing another step. The selection of the right moment for throwing the missile at the target depends on two factors (besides the given velocity of the missile), the length of the line BA and the velocity of the train.



Now, in the abstract, there is no ultimate point

B^n , beyond which it would be useless to despatch a missile, whatever the velocity of the train. For there is no longest possible hypotenuse, and, according to Euclid, it would always be possible to draw a straight line from A to any point on the B line, however remote. We note, however, two facts of interest, (1) that, the greater the length of BB^n , the larger is the area of the triangle across which no missile despatched from B^n will pass; (2) that if all trains were of the same velocity (or if none could exceed a certain finite velocity), the position and significance of B^n would be sharply defined. Its actual distance from B would then depend on the length of BA . The further BA was extended, the more remote would be the position of B^n , regarded as the point from which a train with a given velocity must discharge its cargo for A . Or, on the supposition of trains with different velocities within the finite maximum, only the slower ones would be able to send a missile to A from any point between B^n and B .

Next let us adapt the argument to the case of a revolving circle.



Here the line AB^n is constructed on the same principle as in the preceding diagram. The line CB^{2n} is parallel to AB^n , and the point B^{2n} is meant to stand in the same relation to C as B^n to A . If we assume, as before, that all trains are of the same velocity, or that there is, at least, a maximum velocity, then, the greater the length of the diameter BC , the larger will be the area within which no bodies (*e.g.* spherical masses) will be touched by missiles travelling along the line $B^{2n}C$.

And now let us pass again, so far as possible, from Euclid to the geometry of motion. The circle, I fear, we must retain for the sake of simplicity, but instead of lines like AB^n and CB^{2n} , we should have velocity-curves with their length determined (in Euclidean language) by the length of the other two sides of the triangle. Any such point as B^{2n} must no longer be pictured as a point on a straight line, nor even as a point on the circumference of a rotating circle (for the circle is too Euclidean), but as a point somewhere in the path of a curve, the exact nature of which cannot be indicated. To represent the physical universe as a system of concentric circles is easy enough, but it is much harder to form any distinct image of a vast system of regular velocity-curves forming a totality which we can only describe rather

negatively, as a non-Euclidean sphere. Yet, if we suppose the distribution of 'particles' to be effected in some such way, and pass over the awkward question, why any mass like the earth or the sun should ever be formed at all, we can understand how only the curves within a certain range would pass through any particular mass. Each mass would be surrounded by a kind of precinct proportionate to the length of its 'diameter,' and so would have its own *flammanitia moenia mundi*, from beyond which no traveller would be admitted. At the same time, the principle (whatever it may be) which accounts for the formation of one solid mass would operate perpetually, and new masses would continually be formed by what (in the language of our metaphor) may be called bad shots at the old ones. And what an eccentric universe it would be! Incidentally, too, we find here another possible suggestion about the velocity of light. For the figure denoted by c (though the actual number of kilometres may still be only an index to our own limitations) may be an expression of the maximum velocity of any traveller than can arrive at the earth.

Quite apart from this rash excursion into the cosmos, the argument of this chapter leaves us with one important corollary. For as soon as we understand that the geometry of motion

entirely excludes the Euclidean straight line, we perceive that the path of every ray of light coming to the earth from the sun or elsewhere *must* be curved. There is not the smallest need to drag in 'gravitation' as a mysterious force. As soon as one body is regarded as being in motion relatively to another, it is as certain as in the case of the orange flung from the train at the target on the bank that every traveller from one to the other must advance in a curve. Put a candle on a little table and walk round it. You may then look as 'straight' as you please at the flame, but every ray will travel to your eye in a curve, and the form of the curve will vary according to your distance from the candle and the pace at which you walk.

CHAPTER VII

NON-UNIFORM MOTION AND GRAVITATION

THE strongest objection to discussing non-uniform motion is that we cannot allow the existence of any such thing. Our whole enquiry will be deprived of whatever value it has if we fail to hold fast to the direction in which it has moved. Potentially, at least, we rejected 'non-uniformity' as soon as we began to reflect critically on the swimmer in the stream. The very notion of it is, in truth, an indefensible compromise between the abstract and the concrete. Either we must persevere in the abstraction, ignoring superficial variations in favour of the underlying constancy of direction and velocity, or we must endeavour to impart to the whole career of each particular body the kind of inner unity that belongs to an individual life. But since the latter course would clearly be inappropriate to the science of velocity, we must stick to the other alternative, and must learn to look unmoved, for instance, upon the crash of one train into another, when all the uniform velocities and directions seem to be lost in a bloodstained heap. To the dis-

passionate eye of science all remains as before, placid, constant, unwavering, without change of direction or speed. King and queen, pawn and bishop, knight and castle, one and all preserve their studied modes of velocity, though all be tossed together and packed in a box. The physical world, indeed, is no ordinary chessboard. Amid the intricate convolutions of spherical rotation, it is not surprising if we fail to pierce through the bewildering variety to the principle of uniformity beneath. Things jerk and hop and stagger and swerve, changing gear and direction every other moment, or sinking, to all appearance, inanimate and prone. Yet, before we decline to look upon even the most rakish of progresses as a continuous and orderly career, we are bound at least to ask ourselves seriously whether 'non-uniform' motion can convey to us any meaning at all, except when we impose a test of uniformity determined by some arbitrary point of view. A movement may last, as we say, the merest fraction of a second, but can it be non-uniform while it lasts? Each fresh velocity-curve absorbs and unifies all its ancestors, and each of them preserves its being no less perfectly than the swimmer, who continues to swim his normal number of yards per second when you cast him into the rapids of Niagara

or the maelstrom of a boiling sea. No scientific conception can be grasped without an effort of imagination, or retained without a certain obstinate pertinacity. Consider for a moment the simple dogmas of arithmetic, which few so much as offer to question. The assertion that $2 + 1 = 3$ arouses no uproar, though only heaven knows what it means. It does not mean, presumably, that two veterans and a baby are as strong as three musketeers, or as heavy as any three of the planets; and if none care to formulate such criticisms, that is not because the propositions of arithmetic square too closely with commonsense, but because they are too absurd to discuss. In the mere suggestion, therefore, that 'non-uniform motion' is only a name for the complexities of analysis and calculation, there is nothing more startling than in the analogous thesis, that the method of counting your chickens is the same before and after they are hatched.

Rotation seems always to provoke a special curiosity, if only on account of its near relationship to the phenomena of gravitation and weight. Here again we are obliged to consider what kind of questions it is reasonable to ask, and what kind of answers may fairly be expected. To provide a sufficient explanation, even in a physical sense, of the origin of rotary

motion in general, or of the earth's particular style of revolution, is a task entirely beyond the scope of an hypothesis which deliberately omits all such concepts as 'energy' and 'force.' On the other hand, when a special difficulty is made about rotation or, at an earlier stage, about the Euclidean circle, the appropriate comment is that, not the circle, but rather the 'straight' line, should be questioned and pressed to exhibit its *raison d'être*. Geometry, like most sciences, has usually been arranged in such an order as to display to beginners its ostensible elements, just as children are introduced to the alphabet before they come to the reading of words. Thus the point and the line are presented as milk for babes, even though it is a little disconcerting for the infants to hear that the first has no magnitude, and the second only length without breadth. Such pills we are induced to swallow by respect for our elders, not less conspicuous, some of them, for breadth than for length. The straight line, too, has an air of simplicity. It resembles, if not all roads, at least the roads of the Romans, and seems to express the idea of direction more plausibly than the rim of a plate. Yet this habit of putting elements before compounds has its disadvantages for the education of the mind. It makes us forget the old and valuable maxim,

that the whole is naturally prior to the part. Points, lines and surfaces are abstractions from the solid; the elements of figures are derived from the figures themselves. On the same principle, the sphere alone can serve as a quarry from which blocks of every shape can be cut. And so, when we begin to review the species of motion, we must call to mind, what has been discovered already, that the straight line is peculiar to the geometry of rest. Rotation of some kind (not necessarily in a circle) we are entitled to take as original; while straight translation and all intermediate varieties should be allowed to exist only on sufferance, as abstractions sometimes useful to the understanding or as concessions to our practical needs. Hence, if gravitation be rightly described as a phenomenon of rotation, there is good reason for adopting the provisional hypothesis, that gravitation and motion are one.

Not long since we were discussing, in semi-Euclidean fashion, the effect of throwing missiles at a centre that declined to be hit. We guessed how a solid mass might thus come into being, with a shape not far removed from a sphere. Now, obviously and rightly, we must decline to produce any reason for locating a centre or nucleus 'here' rather than 'there,' and for leaving vast, yawning gaps between.

To the recorders of astronomical history, to the bold explorers of magnetic fields and poles, to the alchemist still 'cherishing his eternal hope,' and perchance to the adept in sciences as yet unborn, it belongs to search for answers to questions such as these. I suggest merely that the mass of the earth, and other similar masses, may have been formed by collision and cohesion of 'particles,' as they converged along curving radii upon a 'centre of gravity,' as though from every point on the surface of a rotating sphere. If this hypothesis be allowed to stand for a moment, will the meaning of gravitation be simplified, and the fall of the apple excused? The most essential thing, if we would make any progress, is to bear always in mind the constant, uniform velocity, not of light alone, but of every swimmer in every stream. Seize a floating log, as it moves with the sole velocity of the river, and moor it for a thousand years to the bank. Set it loose then, and what can it possibly do but proceed in its vehicle, as though the millennium had never intervened? Would you expect it to be cured of its habits? Were you now to shove it upstream for a mile or an hour, would you marvel if it returned as before? And is it any more (or any less) wonderful, if the stone projected 'upwards' falls back on its carriage, the earth? The stone

was on the earth for no other reason than because it was going that way. To be sure, it was not a stone when its substance arrived here some millions of years ago; but that, once more, is a fact beyond the range of this enquiry. Why one thing is a jagged flint, another a cabbage; why fire kindles straw, and is quenched by water; why anything is what it is, and does what it does,

*limus ut hic durescit, et haec ut cera
liqueescit,*

it is not for the science of velocity to decide. All we are justified by our speculation in affirming is that, when one body lies or presses on another, it does so because the other was in the way. The passenger was interrupted in his journey, and is waiting an opportunity to proceed. The mocking semblance of opportunity is provided when one drops a stone over a precipice, or tosses it into the air. It struggles, so to speak, towards the old direction, but once more collides with the earth.

For the better expression of the problem I must revert once more, and for the last time, to the orange flung from the train. Suppose a student of oranges were resolved to map the course of a particular specimen on a particular day, as a nurse takes her patient's temperature at intervals, and registers the fluctuations on a

chart. The orange leaves London in the pocket of a traveller, who takes his seat quietly in the train. The speed of the train advances slowly from zero to its standard 60 miles an hour. Now and then, perhaps, the passenger takes the orange from his pocket, plays catch with it, or bounces it gently against the side of the carriage, misses it presently and drops it on the floor. In due course the moment for displaying his marksmanship approaches; the orange shoots from the window at right angles to the train, sails smoothly along its new velocity-curve, and strikes the human target on the cheek. Even now its vicissitudes are not finished. It rolls down the bank at a modest pace, and reposes at last in the gutter, where it merely rotates about the axis of the earth and circles with the earth round the sun. The student, I fear, will find it a tortuous business to sketch on his drawing-pad the story of so eventful a day. Effects such as 'the portrait of an orange travelling with several velocities and in several directions at the same time' seem to call for a new post-futurist technique.

To maintain in the face of such complexity that the velocity and direction of the orange are constant is no mean challenge to faith. Yet, in a sense, it is all perfectly simple. We have seen already how, in one brief act of its drama,

it swam its 10 yards a second in the direction of its leap from the window, notwithstanding the interference of the river. So again, if we choose to start from the moment it began to pass along the line from B^1 or B^2 to A with a certain velocity, we should rightly argue that neither that velocity nor that direction were extinguished when it struck the mark and rolled down the hill. Or, had it chanced to stick to the target, we should properly regard it as waiting to proceed as before, whenever the earth thought fit to get out of the way. And so, when we remark, as probably we should, that the rolling into the gutter had something to do with 'gravitation,' with what pretence of reason do we assume that this particular episode confronts us with a new kind of problem? If we pick the orange up now and fling it into the air, down it will come again a few feet or yards away. But do we require a mysterious 'attraction' to account for its fall, any more than we require a mysterious 'expulsion' to account for its departure from the hand of the thrower? Or must we talk about 'action from a distance,' any more than when a log is observed to float with the stream from one point on the bank to another some miles away? Why anything goes anywhere, with any velocity, I have not the slightest idea; but why 'gravitation'

should be put in a class by itself is almost more hard to understand. Far simpler is it to accept the hypothesis that gravitation is the original, constant velocity and direction of all things. What demands explanation is not so much the persistence of the original as the multiplicity of apparent deviations. To map and interpret the whole is infinitely difficult, but the task does not differ in principle from the analysis of one velocity-curve like the flight of the orange to the bank. In that particular case we can, nay, we *must*, confess that its uniform direction and velocity are maintained. Just so do all things gravitate towards a 'centre of gravity,' though some are now welded together into our earthly residence, others scattered through innumerable stars. Hence to doubt whether light has weight, unless by light you mean a phenomenon of consciousness, is almost as strange as to doubt whether a passenger will reach the ground when he leaps from the window of a train. If light moves at all, it gravitates; if it gravitates, it can be weighed in the scales.

Gravitation, then, is the 'matter' of motion, comparable to the original stuff and substance of things, which physicists have striven to discover and describe ever since Thales first declared that the history of the world was writ

in water. As 'matter' never appears in its purity, but only in things of determinate quality, so does gravitation, the original velocity and direction, never reveal itself but in the monstrous complexity that baffles the understanding. When we pick up a stone or a grain of dust, we hold in our hands the history of a million velocities and directions, not one thousandth part of which can ever be known and discounted in our search for the original drift. When we think of the world as a whole, imagining it, perhaps, as a sphere rotating with endless regularity, it is barely possible to impute to 'direction' anything more than a local and arbitrary meaning, chosen to suit the description of a certain group of phenomena. We say, for example, that a body falls (with uniform acceleration) 'more swiftly' as it approaches the earth. But why not say that it moves more and more 'slowly' in the opposite direction? Thus does a swimmer, with his face to the stream, move more slowly (as they say on the bank) in one direction, as the stream urges him more swiftly in the other. Meanwhile, his motion, as by now we should understand, is neither swifter nor slower, but constant and uniform. Local prejudice must not be allowed to obscure our vision. Though we cannot hope to unravel the whole

tremendous skein, it will be something if we can grasp in our calculations some of the more prominent directions and velocities, as, for example, the rotation of the earth. When the stone is cast 'upwards,' it makes a fresh zig-zag on the chart of its movements, but still in relation to the line of normality. Throw a piece of paper from the window of a moving train, and, as likely as not, it will come back through your own or a neighbouring window. Throw an orange after it, and it ceases, superficially, to travel in the line of the train at 60 miles an hour, nor does it appear to depart, with its own speed of 10 yards a second, at right angles to the direction of the train. Yet analysis proves that it does both at once. Repeat the experiment, with the earth for your train and the complex of rotation and orbital velocity for your river, and why should you stand aghast and bewildered when your missiles do not fade away beyond the horizon of sight, *ripae ulterius amore*? Or once more, when a billiard ball is flipped away with a 'screw' on it, it runs forward a few inches, and then returns to your finger or near by. Think, then, of the portentous 'screw' or 'side' on a stone when you hurl it up into the air. Is it wonderful if it cannot instantly divest itself of all its existing velocities and directions, and

flash back, for your eyes to behold, into its original curve? A bare glimpse of the past, we may fondly imagine, is afforded us when a body is allowed to fall through a vacuum; but, whether that be so or not, the only reasonable hypothesis is that a body, in so far as circumstances allow it, tends always to resume its original curve. But that curve it was that brought it to the earth. What the original velocity may have been is, perhaps, only guesswork. Yet if c be the greatest velocity, or the greatest we can measure, within the precinct round the earth, the most sane conjecture will be that the velocity of all things, at least within one particular zone, is most nearly gauged by the velocity of light.

The riddle of beginnings is beyond our outlook. Already it has been freely admitted that no reason can be offered for the formation of a nucleus or centre at any particular 'point' within the whole. Given the existence of one such 'centre of gravity,' the image of a target with inner and outer rings may help us faintly to picture how a system of sun, planets and satellites might arise. In place of bullets arriving, some in the bull's eye, some in inner, some in outer rings, we have to think of velocity-curves, some of which pass through the earth (the sun, or whatever it may be), others

near it, others farther and farther away. Thus the approximations to (or deviations from) any given centre will vary in degree, and the 'gravitation' of one mass to another may indicate the degree of proximity to a certain direction-curve, or it may depend on the actual interception of 'particles' by a mass which stood in their way. The secret, it is obvious, will never be revealed until it is known how any centre or nucleus is formed. Meanwhile, the greatest practical difficulty lies in the attempt to keep any hold on the meaning of 'direction' when we renounce the straight line altogether. We can fix our own directions by local interests and certain terrestrial phenomena; we can even get as far as mapping the correlative motions and varying positions of the heavenly bodies within the range of our observations; but what 'direction' means in relation to the whole physical universe, dimly pictured as an orderly whirlpool of spherical rotation, who is prepared to say?

The sun may 'attract' the earth, but all such terms as 'attract' and 'repel' are excluded by an hypothesis which confines itself to the notions of velocity and direction. Newton's magnificent attempt to express in a simple formula the relation between all bodies is, as everyone knows, not an attempt to say what

gravitation *is*, or why there should be any such thing. For that reason it is compatible with a dozen different explanations, and does not stand or fall with any one of them. The history of science has shown a hundred times and more that phenomena can be sufficiently described, and even accurately predicted, while the supposed basis of theory is radically unsound. Calculations may be verified to a nicety, and verification may yet be very far from proof. It may now be the destiny of Newton's formula itself to become only a treasured curiosity in the museum of bygone ideas. None the less true will it be that by the aid of that formula a thousand facts otherwise inexplicable were 'explained.' There is a sense, therefore (and no disparaging sense), in which even the calculations of a Newton are less important than the conceptions of time, space and motion, on which they are thought, perhaps wrongly, to depend. So it is with the brilliant predictions of Einstein, a stroke of genius not unworthy to be compared with Newton's. For my own part, I must decline to allow a genuine connection between the now famous astronomical results and a theory of 'relativity' not adequately conceived. For the whole doctrine of relativity is fallacious, so long as the lesson of the swimmer and the

passenger is read crookedly, so long as the 'addition of velocities' is a cause of confusion, and so long as the 'constant velocity of light' is considered apart from the fact that the velocity of *all* things is constant. It is probable (though I have no adequate means of judging) that the phenomena so brilliantly explained by Einstein will prove to be complicated with optical facts, such as we have discovered in the error of the bank-observers, when they suppose a swimmer or a passenger to be moving along a line that he studiously avoids. Let us beware, however, of the dubious words 'subjective' and 'illusion.' Let us prefer to say, for example, that certain very elusive facts may be revealed to observation only when the sun is totally eclipsed. If one day, through some abnormal condition of the atmosphere (perhaps through nothing more extraordinary than a mist), the observers of one swimming along what they regarded as a straight line, some 60 yards in length, were suddenly to see him travelling along a hypotenuse of 75 yards, what would be likely to happen? Would they shout at him and bid him return to the proper line? Would they be content to dismiss it as 'optical illusion'? Or would some passing critic succeed in convincing them that, for once in a way, they were looking with open eyes at the *fact*?

This brief survey is now completed. The argument has been wholly physical in character, and must not be mistaken for anything but a discussion within the bounds of a very narrow hypothesis. Every science is created by an act of wilful abstraction; one might almost say, by a determination to ignore most of the facts. Every science, therefore, destroys its own value as soon as it forgets its own appointed limitations. The world was not created with a pair of compasses, not yet by the inverse calculation of distances and squares. Skimming lightly over the surface of the universe, mathematics and physics may gather up many fragments of knowledge for the instruction and use of mankind. Only when, confounding appearance with reality, they suppose themselves to have reached the heart of the mystery, does the resemblance between these distinguished sciences and nonsense become almost perilously complete. Yet the mind of man is busy and unquiet. Among the many children of Pythagoras not a few have learned, by much pondering of line and number, to read in the language of shadows a meaning profound and true. So once Plato, grown weary of 'hypotheses,' aspired to climb by the crooked stairway of dialectic to sublime contemplation of the good. So, long afterwards,

the greatest of his disciples, Plotinus, dared to launch his spirit on that hazardous voyage. Forsaking its earthly lodging, it sped undaunted through regions of air and fire, away past sun and planets, beyond the uttermost multitude of the stars; till it came at last to the awful silence, where sits throned in unfading majesty the eternal, the inviolable One.

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